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REVIEW AND CRITIQUE OF THE VECTOR-2 COMBAT MODEL

Alan F. Karr

December 1977

INSTITUTE FOR DEFENSE ANALYSES
PROGRAM ANALYSIS DIVISION

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is a review and critique of the VECTOR-2 theater-level simulation model of conventional ground-air combat. Aspects of the model analyzed are organization and general structure, environment, resources and logistics, command, control and communication processes, attrition in ground combat, attrition in air-combat, movement of ground units and aircraft, intelligence and target acquisition, and computer-related aspects of the model. The paper emphasizes

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identification and understanding of assumptions underlying the mathematics of the model (and especially of the attrition processes), with the objective of fostering better understanding of all models.

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I. INTRODUCTION

VECTOR-2 is a deterministic simulation model of theater-level bilateral combat not involving biological, chemical, or nuclear weapons. It is intended for making net assessments, general purpose force tradeoff analyses, and studies of strategy and tactics. Our reaction to it is generally positive: it possesses impressive detail and flexibility and seems to be based on reasonable underlying assumptions.

This paper is a review and critique of the VECTOR-2 theater-level combat model and represents a continuation of previous work of the author reported in the following papers:

1) *On the VECTOR-1 Combat Model*, P-1181, Arlington, VA: Institute for Defense Analyses, 1976;
2) *On the Lulejian-I Combat Model*, P-1182, Arlington, VA: Institute for Defense Analyses, 1976;
3) *On the CONAF Evaluation Model*, P-1210, Arlington, VA: Institute for Defense Analyses, 1976;
4) *On the IDAGAM I Combat Model*, P-1265, Arlington, VA: Institute for Defense Analyses, 1977.

These papers are referenced as [8], [9], [10], and [12], respectively; the reader wishing to compare and contrast the various models is urged to consult these references. Also relevant, and complementary to the author's papers, is *Comparison and Evaluation of Four Theater-Level Models: CEM IV, IDAGAM I, Lulejian-I, VECTOR-1*, L.K. Walker, M.S. Higgins, and W.G. Svetlich, R-299, Arlington, VA: Weapons Systems Evaluation Group, 1976, referenced as [20].

Much of our analysis treats mathematical structures and assumptions underlying them, with further emphasis on attrition

processes in particular. While this concentration reflects biases, tastes, and capabilities of the author, it is also a serious attempt to facilitate understanding and comparison of theater-level combat models. Classical methods of evaluating and comparing models--such as using verifiably accurate data as inputs and then comparing outputs with empirical data, or testing by means of a controlled scientific experiment--are, for theater-level combat models, either impossible or extremely impractical. Moreover, a controlled experiment very likely would show only that no model is fully compatible with physical reality and hence would not provide a basis for making a choice among models in situations where one must be chosen for some particular analysis or study. An experiment might, however, show some models to be manifestly less realistic than others.

By treating underlying assumptions and mathematical structures we are able to describe some limitations of the different models, which is an essential prerequisite to using any of them; at least gross misuses may thereby be prevented. Moreover, once assumptions are understood, the model user has one rational basis for making comparisons and choices among models. Hence, even though some comparative comments below are stated as judgments, the objective has been to elucidate differences among models so that all may understand them better.

The remainder of this paper contains many references to the "three comparable models"--the CONAF Evaluation Model, the IDAGAM I Model and the Lulejian-I Model; we have not compared the VECTOR-1 and VECTOR-2 Models. The author's intention is to contribute to the common intellectual foundations of all the models and to greater understanding of the models both individually and relative to one another; criticisms are in this spirit throughout.

In preparation of this paper, our main source was the report [14], although [16] was frequently used to clarify points

not treated clearly in [14]. The reader is urged to consult [14] and [16] in conjunction with this paper.

2. ORGANIZATION AND GENERAL STRUCTURE

The VECTOR-2 model is structured as a feedback control system or, more precisely, as two interacting feedback control systems--one for each side. Operation of these systems, each of which contains several subsystems that are themselves feedback control systems, is described in more detail in Section 5. In broad terms, each decision maker compares perceived and desired states of the combat and takes actions designed to make the perceived state more like the desired state according to some criterion. The evaluative portion of the model, which computes attrition and movement, serves to define the actual state of the combat, of which the decision makers possess only imperfect knowledge. It should be noted that the property of being a "feedback control system" is not unique to the VECTOR-2 model, although as a means of explaining the model it is unique to the VECTOR-2 documentation. The three comparable models are also feedback control systems.

Figure 1 depicts the way in which model structure determines evolution of the combat over one time period. As usual, the two sides are called Blue and Red.

In representation of time, the VECTOR-2 model appears to differ significantly, although not necessarily to the extent implied by [14], from the CONAF Evaluation, IDAGAM I, and Lulejian-I models. The model incorporates six nested levels of fixed time steps which have the following proposed values in [14]:

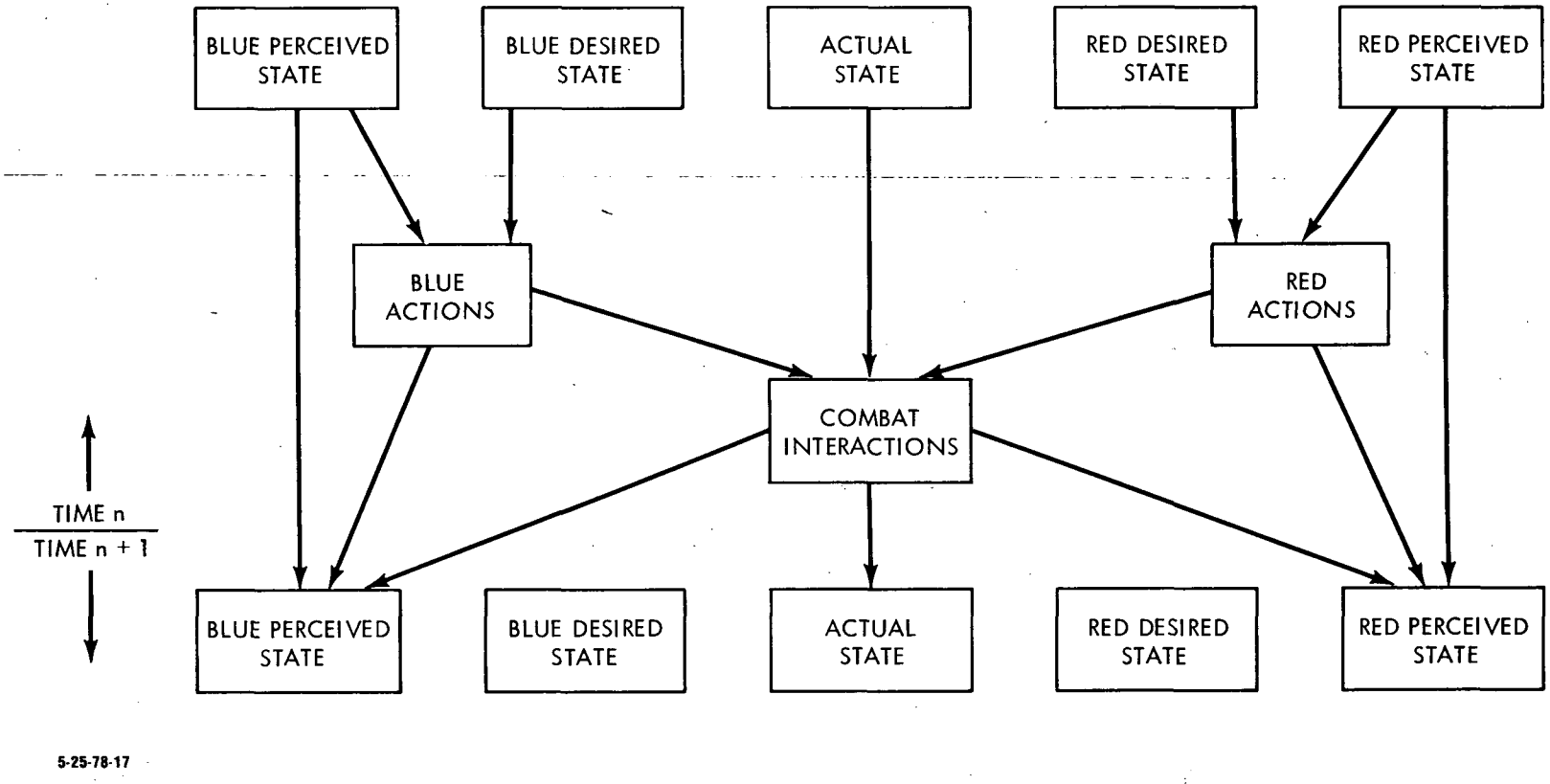

Figure 1. CAMPAIGN EVOLUTION IN VECTOR-2
(Schematic Representation)

LEVEL	PROPOSED INTERVAL
1	30 seconds
2	3 minutes
3	15 minutes
4	1 hour
5	6 hours
6	24 hours

The tabulation given above implies that Level 1 calculations, which represent front line combat using methodology described in Section 6, must be performed 2880 times per day of simulated combat. This may be a significant computational burden; cf. Section 10 for further remarks. In [16] rather larger time steps are proposed.

In addition to the set of nested time steps, the VECTOR-2 model contains a dual method of event scheduling. Events of long-term influence, such as force arrivals and aircraft mission assignments, take place at regularly spaced, artificial intervals, i.e., at the epochs of an appropriate time step level. On the other hand, events of short-term and spatially limited influence, such as maneuver unit close combat engagements, local movements of maneuver units, and arrivals of support fire are asserted to be scheduled "dynamically." This appears to be more an interpretation than an actual property of the model inasmuch as the computational structure of the model seems entirely time sequenced (cf. the flow chart on pp. 1-30 to 1-33 of [14]). Possibly what is meant is that events of short-term influence are assigned nominal times of occurrence, but that effects of these events are assessed at (for example) the next succeeding Level 1 time step.

Even so, use of extremely small time steps makes VECTOR-2 significantly different from the three comparable models in two respects. Obviously there is a greater level of detail, and hence possibly greater realism and flexibility, in VECTOR-2.

Less obviously, but perhaps more importantly, the fine time steps mitigate many of the problems with interaction sequencing that occur in other models because of arbitrary, albeit necessary, choices of orders of interactions. Over small time intervals, attrition is sufficiently small that spurious effects of such choices probably are negligible.

Somewhat surprisingly, given the level of detail of the model, there is no capability for day/night differentiation of effectiveness parameters and, indeed, essentially no capability to represent night time combat. The times when combat begins and ends each day are specified by input; it appears from [16] that one should not allow these to include the entire 24 hours, but probably only daylight hours.

Combat-related processes represented in VECTOR-2 may be put into the following categories:

1) Combat interaction processes (attrition)
2) Command and control processes
3) Intelligence and target acquisition processes
4) Communication processes
5) Logistics processes
6) Movement processes.

Each class of processes is discussed in detail in an appropriate Section below. The listing that follows gives the time step levels on which various process effects are calculated.

PROCESS	TIME STEP LEVEL
1) Theater-level processes	
a) Resource arrivals	6
b) Planning functions	6
2) Sector-level processes	
a) Resource arrivals	6
b) Intelligence processes	
• for theater-level and sector-level planning	6

PROCESS	TIME STEP LEVEL
• for front line maneuver units	3
• for other forces	4
c) Planning and mission and force allocations	
• for front line maneuver units	3
• for other forces	6
d) Reorganization, replacement, and resupply	6
e) Maneuver unit combat and movement	1
f) Target acquisition for support fire	
• for maneuver units as targets	2
• for other combat resources as targets	3
• for all other targets	5
g) Support fire allocation and effects	
• against maneuver unit targets	1
• against all other targets	3
h) Aircraft movement and interactions	1
i) Miscellaneous attrition and repair processes.	3

As a whole, the structure seems reasonably general and flexible. In particular, if it is possible to make all time steps the same size, in which case the nesting serves only to determine the order in which computations are performed, then VECTOR-2 could be reduced to a fixed time step model like the three comparable models. Note, however, that a change in the time steps requires concomitant changes in rates that are inputs to the model. Nonetheless, VECTOR-2 is structurally more general than the other models.

3. ENVIRONMENT: GEOGRAPHY AND WEATHER

In this Section we describe environmental aspects of the VECTOR-2 model, namely battlefield geography, terrain, and weather. Representation of these three aspects of combat is uneven: geography is unusually detailed and sophisticated for a theater-level model, with front-line maneuver units located in a real-space coordinate system. Terrain is represented parametrically in essentially standard fashion. Weather, one of the most important exogeneous, stochastic influences on combat, must be specified by input and affects evolution of the combat only through various indices of visibility and trafficability, as we will discuss in somewhat more detail. Weather intelligence is included as user-input five day forecasts for each side.

We shall describe fixed battlefield geography first in schematic form and then in terms of actual physical coordinates, after which "floating" geography, which is organizational and moves as forces do, will be treated. Figure 2 represents schematically the fixed battlefield geography.

Sectors are the largest geographic units in VECTOR-2 and are important in that combat cannot occur across sector boundaries; indeed, sectors are decoupled in the model to the extent that, except for theater-level calculations, only one sector at a time is treated in the core of the computer. At most seven sectors are permitted. From the standpoint of combat assessment, *combat arenas* are the principal geographic units in VECTOR-2; maneuver unit engagements do not extend across combat arena boundaries. A combat arena must be large enough to contain

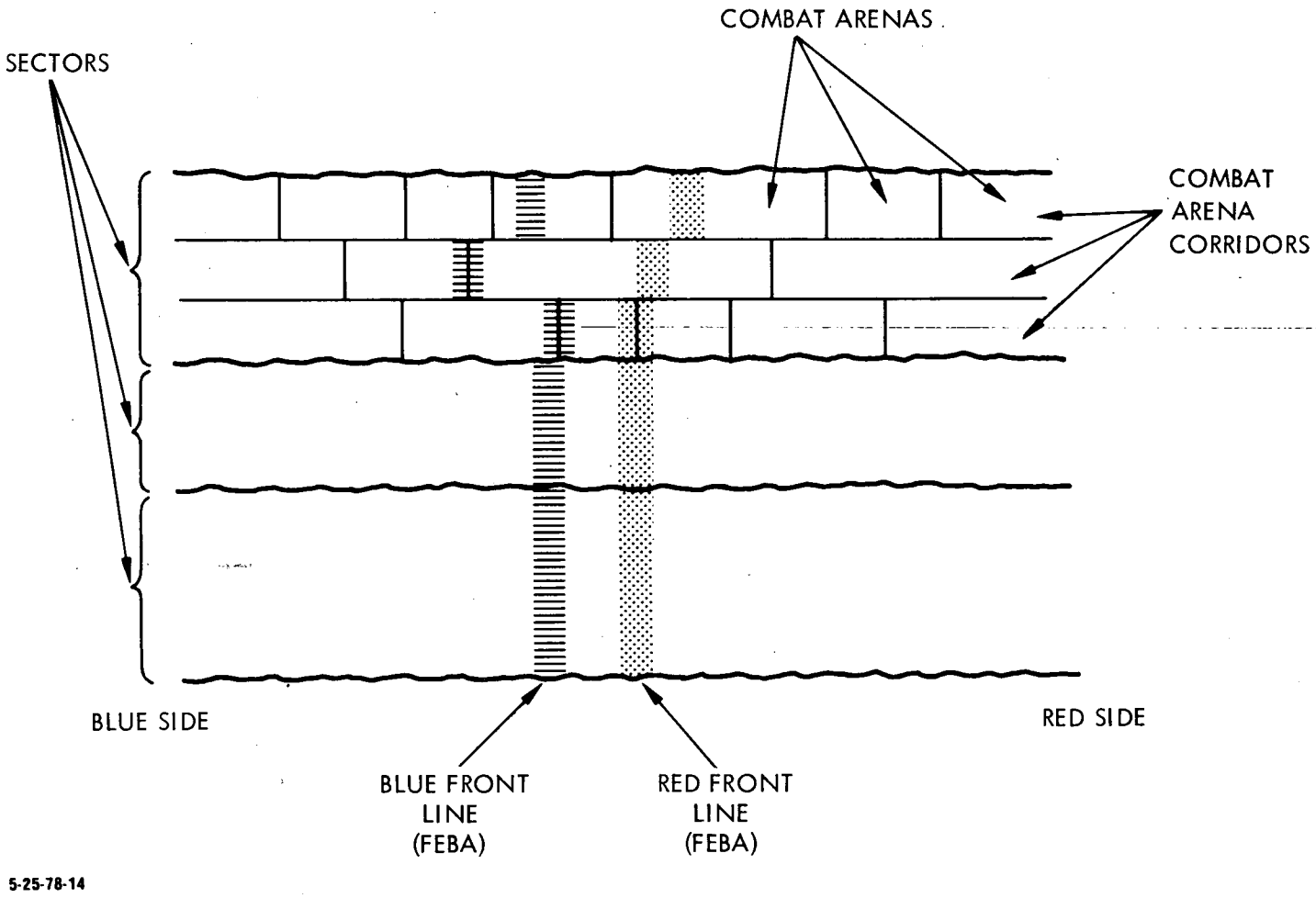

Figure 2. SCHEMATIC BATTLEFIELD GEOGRAPHY IN VECTOR-2

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a battalion-sized independent defense; its end boundaries represent terrain features (e.g., rivers) or changes in terrain type. It may contain arena-wide defensible positions, which are equally spaced but not closer than the range of direct fire weapons. The latter requirement ensures that opposing forces cannot enter a direct fire engagement when both are at defensible positions.

Combat arena corridors are strips of combat arenas that are adjacent front-to-back and run parallel to sector boundaries, as shown in Figure 1. Also as shown there, a combat arena has only one adjacent arena to its front and only one to its rear. A sector may contain no more than fifteen combat arena corridors.

Unlike those of comparable models, the FEBA in VECTOR-2 consists of two curves that represent the front lines of the Blue and Red sides, respectively. The FEBA need not coincide with the schematic geographical boundaries in the model; it may even run through a combat arena as shown in Figure 2 (in other places the FEBA does coincide with arena boundaries). In each combat arena corridor, FEBA position is schematically constant and the opposing front lines are separated by a positive distance. FEBA position over a sector need not be constant. As discussed in Section 8, FEBA movement is computed within the model from physical movement data and not, as in CEM, IDAGAM I and Lulejian-I, from artificially devised tables or systems of equations.

In the three comparable models, schematic geography and actual geography are virtually indistinguishable; in VECTOR-2 schematic geography is truly schematic. The model contains an explicit coordinate geography in terms of which aircraft and maneuver units are located. An example is given in Figure 3. Where, for simplicity, only the topmost sector of Figure 2 is repeated. Clearly, specification of combat arena boundaries entails significant effort.

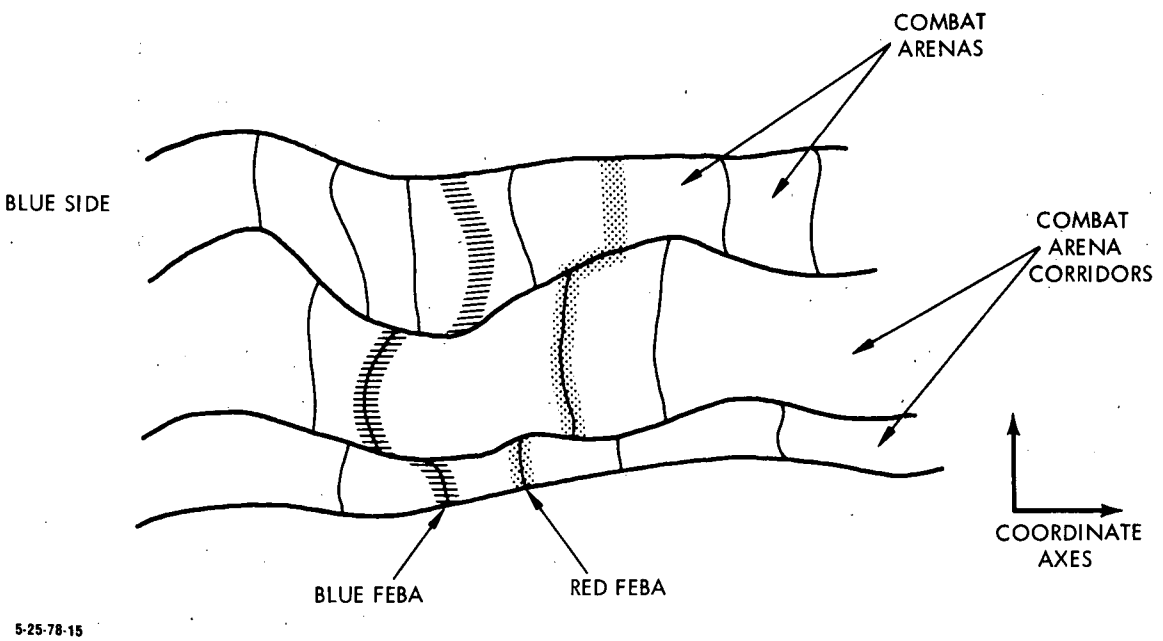

Figure 3. COORDINATE BATTLEFIELD GEOGRAPHY IN VECTOR-2

The geography discussed above is representative, either schematically or exactly, of actual battlefield geography; in addition VECTOR-2 incorporates an organizational geography that moves as the FEBA does and serves to locate less important resources and in resource allocation. Figure 4 depicts organizational geography; for simplicity the organizational geography is superimposed on the schematic geography of Figure 2 and shown only for one sector and only for the Blue side.

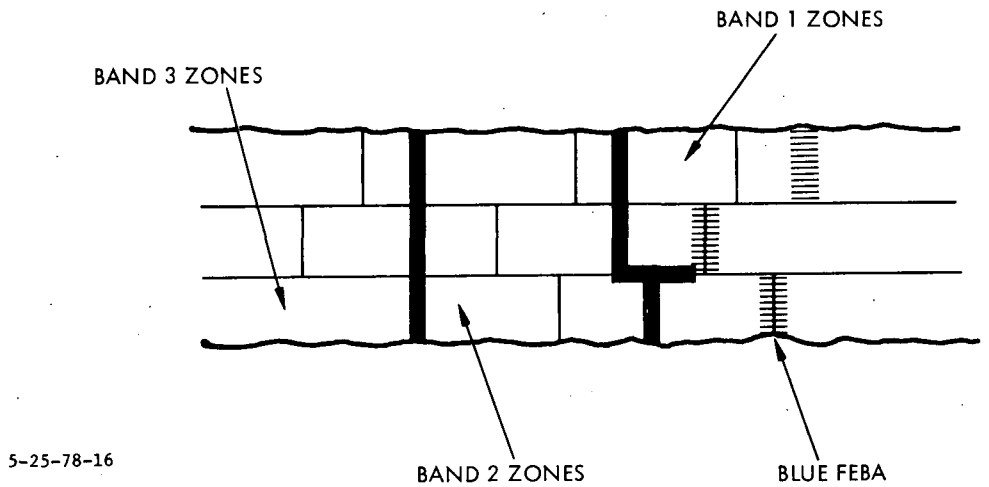

Figure 4. ORGANIZATIONAL BATTLEFIELD GEOGRAPHY IN VECTOR-2

Floating, organizational geography is represented in VECTOR-2 by means of *zones* that move with the FEBA and serve to define various rear regions. Lateral boundaries of a zone are specified combat arena corridor boundaries, while the back boundary is a prescribed distance from the front boundary. Four bands of zones are permitted. There need not be symmetry or alignment of zone boundaries on the two sides or in different corridors on the same side.

Within this system of geography, front line and reserve maneuver units are represented in actual coordinate locations, as are airborne flights of aircraft. The following combat resources are located only in terms of zones by assuming them to be at characteristic and user-specified distances from the FEBA: field artillery batteries, air defense sites, observation resources, supply depots, and air bases. Particularly for aircraft in flight and air defense sites, this creates asymmetries that do not represent physical reality.

Terrain type in VECTOR-2 is defined by specifying one of six levels of intervisibility (which affects the attrition computations described in Section 6) and one of six levels of trafficability (which influences movement computations); hence there are a total of 36 different terrain types, although presumably not all combinations of trafficability and intervisibility are equally plausible. Each combat arena contains terrain of only one type. Terrain features represented in the model include urban areas, rivers, and one user-defined feature (e.g., mountains); these occur at ends of combat arenas, are at least arena-wide, and require special movements and possible attacks in order to be passed.

Weather is described by specifying one of four levels for each of the following five parameters:

1) visibility for ground-to-ground operations

2) visibility for air-to-ground and ground-to-air operations
3) visibility for air-to-air operations
4) trafficability for ground operations
5) trafficability for air operations.

These parameters are specified by input on a per-sector, per-hour basis and cannot be neglected (for reasons of simplicity or efficiency, for example). Also, air-to-ground visibility and ground-to-air visibility are unjustifiably taken to be the same.

From terrain and weather data the model computes for each combat arena an environmental trafficability index and an environmental visibility index, which are updated each hour. Both indices have parametric effects on attrition and movement computations.

4. RESOURCES AND LOGISTICS

The following is a list of resources modeled in VECTOR-2 and missions those resources may undertake:

1) Ground Forces

a) Maneuver units

Resources: 11 weapon types, personnel of 1 type, minefields

Missions: Pursue withdrawing enemy, advance unopposed, counterattack, attack/defend in urban area, attack/defend at user-specified terrain feature, attack/defend at river, attack/defend at normal defensive position, delay, withdraw under attack, withdraw not under attack, regroup, be inactive.

b) Artillery and mortar forces

Resources: 5 weapon types, personnel

Missions: pre-engagement fire, final protective fire, disengagement fire, fire at acquired targets

c) Air defense sites

Resources: 6 weapon types, personnel

Mission: fire against aircraft and helicopters

2) Tactical Air Forces

a) Aircraft

Resources: 7 aircraft types, personnel, 3 types of shelters (2 of which hold only a specified type of aircraft)

Missions: CAS, escort, intercept, attack of acquired ground targets (reserve units command posts, supply depots, field artillery batteries, air defense sites, air bases, observation resources)

b) Attack helicopters

Resources: one type only, personnel

Missions: fire support, use as front line maneuver unit weapons

3) <u>Observation Resources</u>

Resources: 14 types of sensors

Mission: Acquire targets for fire support weapons

4) <u>Command Posts</u>

5) <u>Supplies</u>

Resources: Ammunition for each type of maneuver unit weapon, land mines, 11 types of ordnance for aircraft and helicopters, aviation POL, ground force POL, one other category of supplies

Virtually all the missions mentioned above are discussed in more detail in Sections that follow and will not be elaborated upon here. It appears that weapon types within a category are distinguished for accounting purposes and only on the basis of differing values of numerical parameters. The numbers of types of weapons seem generally adequate but cannot easily be changed, although the Program Change Monitor discussed in Section 10 makes such an undertaking less hazardous than for the CEM and Lulejian-I models. In IDAGAM I, the process requires only a change in certain inputs.

Command posts seem to serve two roles in VECTOR-2. First, damage to a command post decreases responsiveness of associated forces; second, command posts constitute an accounting device for representing communication delays. See Section 5 for details.

The resources listed above are organized into resource groups with differing combat characteristics. Types and numbers of types of resource groups are the following:

1) Aggregate groups
   a) Artillery batteries: 5 types
   b) Attack helicopter bases: 1 type
   c) ADA sites: 4 types
   d) Sensor groups: 4 types
   e) Supply groups: 3 types
   f) Other groups: 3 types
2) Airfield groups: 3 types
3) Maneuver unit groups: 5 types, including command posts
4) Air groups: 7 types

Organization, force structures, and hierarchies are prescribed by the tactical decision rules of the model; the latter are discussed in Section 5 below. Depending upon the capabilities and resources of the user, force structures and hierarchies are almost unrestricted; very intricate and detailed command structures can be constructed.

Logistics processes in the VECTOR-2 model perform storage and distribution of resources, specifically:

1) Personnel
2) Weapon systems (of types mentioned above)
3) Supplies of 34 types as follows:
   a) 8 types of ordnance for maneuver unit weapon systems
   b) 5 types of ammunition for artillery and mortars
   c) 6 types of ammunition for air defense weapons
   d) 11 types of ordnance for aircraft and attack helicopters
   e) mines
   f) POL for ground vehicles
   g) POL for aircraft and helicopters
   h) 1 general class of supplies.

Supplies enter either the theater or a user-designated sector and may be stored at the sector level or by user-specified

resource groups. Distribution of supplies is computed by tactical decision rules.

With tactical decision rules, the user can construct representations of supply shortages, including the following:

1) zero-one phenomena such as immobilization of resource groups without POL or inability to fire of weapons without ordnance;
2) linear degradations based on supply shortfalls;
3) no effect on resource function, but with supply inventories still computed (and allowed to become negative).

Supplies are depleted by consumption and attrition resulting from enemy fire. In general, supply consumption is a linear function of some state variable that represents the level of activity of a resource. Hence, for example, aviation POL consumption C_A is computed using an equation of the form

$$C_A = c_A(\sum_i s_i) + c_A' p \ , \tag{4.1}$$

where

s_i = length of combat sortie i,
c_A = POL consumption rate, as a function of distance, for combat sorties,
p = number of personnel in sector,
c_A' = POL consumption rate, per time period per person, for noncombat uses.

Similarly, each aircraft sortie consumes a user-specified load of ordnance, which may depend on the type of aircraft and the mission. POL consumption by attack helicopters is proportional to time spent in the air.

For weapon systems in maneuver units, ordnance consumption is proportional to the firing rate; the latter is determined by an equation of the form

$$\rho_i = \rho_i^0 \sum_j f(i,j) \ , \tag{4.2}$$

where

ρ_i = firing rate for weapon of type i,

ρ_i^0 = firing rate when actually firing,

$f(i,j)$ = fraction of time spent firing at targets of type j.

In general $\sum_j f(i,j) < 1$ so that ρ_i accounts for time when the weapon is not firing, whereas ρ_i^0 does not.

The report [14] does not elaborate upon attrition to supplies. Presumably, and very reasonably, the scheme is rather simplistic.

Only limited functions of construction, repair, and maintenance are represented in VECTOR-2, which seems quite justifiable in a model intended primarily for analysis of combat effects only, but not in a model designed to study the participants' capabilities to support their forces. Fixed fractions of damage to ADA sites and command posts are repairable each Level 4 and Level 2 time period, respectively. New air bases can be constructed using tactical decision rules and overrun air bases can be reconstructed, both at user-input rates. Construction of aircraft shelters is effected through tactical decision rules. No repair of maneuver unit weapons appears possible.

5. COMMAND, CONTROL, AND COMMUNICATION PROCESSES

The command, control, and communication processes in the VECTOR-2 model seem sensible and appropriate for a theater-level model, unlike those of the Lulejian-I and IDAGAM I models, which are nonadaptive, inflexible, and over-simplistic, and those of the CONAF Evaluation model, which are so detailed that they overwhelm the rest of the model. Some effects included in VECTOR-2 without great error might be neglected, but the representations seem to depict phenomena of importance in determining the short-term evolution and ultimate outcome of a battle. Moreover, the level of detail is generally consistent with the rest of the model.

In the conceptual sense, the command, control, and communication processes in VECTOR-2 constitute a feedback control system. Each commander is presented with:

1) A *desired state* of the combat, represented by state variables defining mission, objectives, available resources, and so on, values of some of which may have been set by decisions at higher levels;
2) A *perceived state* comprised of intelligence concerning enemy forces, environmental and scenario data, and (possibly out-of-date) information about friendly forces.

Based on the perceived state, the commander undertakes actions intended to change the perceived state into the desired state.

It is claimed in [14] that the model represents decision making "from squad to theater level," but it seems that only for higher levels of command is the representation reasonably complete and accurate. Differences among levels are the size of the system that can be affected, the resources available, and the frequency with which decisions can be altered.

The three decision-related functions may be defined as follows:

1) Command: mission assignment and task force configuration
2) Control: dynamic allocation of resources
3) Communication: information flow and delay.

Command and control are represented primarily by the tactical decision rules described in more detail below in this Section. Each decision is defined by:

1) A point in time at which the decision must be taken
2) Outputs that must be prescribed by the decision
3) Information used as input
4) Method of computing outputs.

Communication is represented by delays in implementation of command and control decisions.

The command hierarchy is user-specified, subject only to the constraint that no unit have more than one direct superordinate; the scheme is advantageous in terms of flexibility but imposes substantial preparations that must be undertaken before running the model. Only maneuver units and their superordinates are included, which is not a serious restriction. Indeed, the lowest level in the hierarchy is the battalion level, at which, appropriately, effects of direct fire engagements are computed. One then questions, however, whether the model really represents decision making "at the squad level." The command hierarchy may change over time, a feature not included in the three comparable models.

To aid in mediation of conflicting resource requirements of proximate maneuver units, an objective overlay exists that characterizes interdependence of objectives in pairs of adjacent combat arenas, contains a representation of terrain, and is used in task force construction, mission assignment,

establishment of forward combat arenas in each sector, and allocation of arena ribbons to brigades or divisions.

With most models, the greatest potential strength is, if not carefully exploited, possibly the greatest weakness and so it is with the tactical decision rules in VECTOR-2. In terms of potential for--

1) providing a systematic and unified treatment of decision making,
2) flexibility,
3) ease of variation, and
4) realism and detail--

the tactical decision rule structure of VECTOR-2 easily surpasses even the decision making structure of the CONAF Evaluation model. But the word "potential" is crucial; to be effective, tactical decision rules must be intelligently and carefully constructed so that the user is certain that effects desired to be represented are in fact represented. If misused or abused, the tactical decision rules render the model and its outputs untrustworthy.

In terms of decision making structure, the model has fixed within it only

1) points in time and command levels at which decisions are required, and
2) for each decision, a minimal set of required outputs, namely state variables whose values must be set before the model can proceed (other state variables can be set or modified if desired).

The user can and must specify, in the form of a FORTRAN subroutine, the method for evaluating required (and any additional) outputs. All state variables may be used as inputs, but care must be exercised to prevent inadvertent changes in values of the state variables.

Inputs to a particular tactical decision rule may be grouped into three classes:

1) Those representing the desired state of the combat (e.g., a mission assignment by a higher echelon)
2) Those representing the perceived state of the combat
3) Data used in comparison of perceived and desired states and computation of outputs.

A simple example is given in [14, pp. 79-82].

The following is a complete list of tactical decision rules in the VECTOR-2 model, along with their subroutine names:

1) *Theater level* tactical decision rules for:
   a) Distribution of theater arrivals to sectors, and distribution of groups and resources among sectors (TRTDIS)
   b) Assignment of missions to sectors (TRTMIS)
   c) Construction of new air bases and shelters (TRBILD)
   d) Assignment of aircraft to air bases (TRADIS)
   e) Assignment of aircraft to air groups and air groups to sectors (TRAGAL)
   f) Setting times for preplanned air missions (TRTIMP)
   g) Setting targets for preplanned air missions (TRAGAZ)
   h) Calculation of rate-based attrition to maneuver units in transit between sectors (TRUKIL).

2) *Sector level* tactical decision rules for:
   a) Assignment of front line responsibilities to maneuver units (TRFRNT)
   b) Reorganization of maneuver units (TRREOR)
   c) Construction of command hierarchy (TRDAD)
   d) Distribution of replacements to resource groups (TRDIST, TRDOLE)
   e) Assignment of nonorganic artillery to maneuver units (TRRART)
   f) Shifts of independent resource groups and artillery among zones (TRSHFT, TRREDO)
   g) Allocation of some air groups assigned to the sector to CAS mission (TRAGAC, TRCASD)
   h) Allocation of air groups to targets (TRACAL)

i) Selection of secondary targets for air missions (TR2TGT)

j) Setting conditions for aborting air missions (TRAAAI)

k) Allocation of attack helicopters from front line divisions to front line maneuver units (TRAHAL)

l) Allocation of air groups with interdiction mission to targets (TRINT)

3) *Battalion task force level* tactical decision rules for:

a) Assignment of missions to subunits of a maneuver unit (TRMISS)

b) Readying a maneuver unit for combat by restructuring its subunits (TRREOR)

c) Allocation of organic attack helicopters from front line maneuver unit to subunits (TRAHGD)

d) Target allocation for air-to-air combat (TRAATA)

e) Specification of choice of action after Phase 1 of air-to-air combat (TRPH1)

f) Specification of choice of action after Phase 2 of air-to-air combat (TRPH2)

g) Determination of possible breakoff by attacking aircraft after each pass on a ground target (TRATK)

h) Determination of maneuver unit close combat decisions, including:

- Choice of activity (posture)
- Calls for commitment or reconstitution of reserves
- Choice of action when confronted by a minefield
- Calls for support fire (TRSITN)

i) Selection of allocation method for normal support fire (TRSPLF)

j) Possible alteration of support fire allocation priorities (TRPRIT)

k) Allocation of support fire to acquired targets (TRFSAL)

l) Calculation of rate-based attrition to target acquisition resources (TRSKIL).

A total of 34 tactical decision rules is required.

For the user who wishes to construct his or her own set of tactical decision rules, significant effort is entailed. However, a demonstration package of tactical decision rules is available from the model developers, which provides a chance to use the model immediately and a convenient starting point from which tactical decision rules can be constructed that represent specific phenomena of interest. Tactical decision rules in this package are described in [16]. Although [14] is not clear about the point, we believe that the two sides need not use the same tactical decision rules.

As noted previously, communications effects in VECTOR-2 are manifested only as delays in transmitting messages between the battalion, regiment, division, and corps levels. Messages may be either priority or nonpriority, but a priority message does not preëmpt a nonpriority message. The model employs the following equations to compute expected delays for messages originating at a given node with a given destination. Let

W_p = expected waiting and transmission time for a priority message, and

W_n = expected waiting and transmission time for a nonpriority message

be the outputs to be calculated and let

λ = rate of message generation at a given node (which depends on whether combat is in progress nearby),

μ = single channel message transmission (completion) rate, and

s = number of channels in communication link.

Then, provided $\lambda < s\mu$, the relevant expressions are

$$W_p = \frac{q}{s\mu(1 - \frac{\lambda}{s\mu})} + \frac{1}{\mu}, \tag{5.1}$$

and

$$W_n = \frac{q}{s\mu(1 - \frac{\lambda}{s\mu})^2} + \frac{1}{\mu}, \tag{5.2}$$

where q is the limiting probability (as $t \to \infty$ and the system reaches equilibrium) that all channels are in use but no message is currently awaiting transmission.

The reviewer cannot derive these equations from the assumptions stated in [14]; at a minimum, the fraction of messages that are priority messages must be specified.

6. ATTRITION IN GROUND COMBAT

In this Section we describe the most complex and sophisticated set of calculations in the VECTOR-2 model: those used to compute ground combat losses. To a significantly greater extent than those of the comparable CONAF Evaluation, IDAGAM I, and Lulejian-I models, the ground combat attrition methodology in the VECTOR-2 model is based on physically defined and measurable input parameters and on environmental conditions. While the other models might represent implicitly in the preparation of data bases such phenomena as existence of lines of sight, range, terrain, weather, and different methods of target acquisition, only VECTOR-2 can do so explicitly and dynamically. The reviewer believes that VECTOR-2 is without question superior to the other three models in representation of this extremely important component of theater-level combat. Further general comments on ground combat attrition computations in VECTOR-2 appear in Section 11.

We begin by describing the VECTOR-2 methodology for assessing effects of engagements between opposing maneuver units, which are typically battalion task forces. The reader uninterested in mathematical details may wish to omit pages 38-47.

Maneuver unit engagement effects are computed individually for each combat arena and calculated once each Level 1 time step (cf. Section 2). Explicit representation of the following phenomena is included:

1) Maneuver unit strengths and coordinate locations
2) Weapon system performance data
3) Environmental conditions: weather and terrain

4) Weapons deployment and movement
5) Mounting and dismounting of infantry from APCs
6) Open- and cease-fire ranges
7) Two methods, parallel and serial, of target acquisition and selection by shooting weapons
8) Existence of a line of sight as a necessary condition for target acquisition.

Every shooting weapon is assigned a list of priorities of targets. These priorities do not represent physically measurable quantities and are judgmentally derived in most cases; to the extent that the priorities influence results obtained using the model, those results may be suspect.

A weapon with *parallel target acquisition* continues to search for new targets while engaging a target and, should a target of higher priority be acquired, will immediately (and instantaneously) switch its fire to that target. A weapon with *serial target acquisition* will, once it engages a target, continue to engage it until the target is destroyed (possibly by another shooting weapon) or the line of sight to the target is lost; during an engagement the shooting weapon does not seek to acquire further targets. The method of target selection, however, is quite complicated and is specified by a sequence of *search cutoff times*. Assume that there are N types of weapons on the opposing side; for a particular type of shooting weapon there will then be N-1 search cutoff times

$$t_1 < t_2 < \ldots < t_{N-1} .$$

If a target of priority 1 is acquired before t_1 (measured from the beginning of an acquisition effort) it will be engaged at once. During this time, targets of lower priorities may become and remain acquired but will not be engaged. If no priority 1 target is engaged before t_1, but a priority 2 target has been acquired and remains visible at t_1, it then will be engaged immediately. If neither of these situations has occurred, the

shooting weapon continues to search for targets. If in the time interval (t_1, t_2) a target of priority 1 or priority 2 is acquired, it will be engaged at once. Should this not happen, any target of priority 3 acquired in $[0, t_2]$ and still visible at t_2 then will be engaged at t_2; otherwise the search continues and any target of priority 1 or 2 or 3 acquired in (t_2, t_3) will be immediately engaged, and so on. The following table summarizes.

Table 1. RULES FOR SERIAL TARGET SELECTION IN VECTOR-2

Time	Priorities of Targets to be Engaged Immediately Upon Acquisition	Priorities of Targets to be Engaged if Previously Acquired and Still Visible
$(0, t_1)$	1	
t_1		2
(t_1, t_2)	1,2	
t_2		3
(t_2, t_3)	1,2,3	
⋮ (t_{N-2}, t_{N-1})	⋮ 1,...,N-1	⋮
t_{N-1}		N
(t_{N-1}, ∞)	1,...,N	

As noted in [14, p. 1-42], important assumptions that underlie the ground combat attrition computations in VECTOR-2 are:

1) Destroyed and undestroyed targets are distinguishable with certainty; a previously destroyed target is never engaged. Destruction of a target is immediately discernible to *all* shooting weapons.
2) All weapons of the same nominal type in the same location are equally vulnerable and equally effective.

3) For a given shooting weapon and target, lengths of visible periods (when a line of sight exists) are independent and identically exponentially distributed; lengths of invisible periods are independent and identically exponentially distributed (possibly with a different expectation) and independent of lengths of visible periods. Visible and invisible periods alternate.

4) Given continuous existence of a line of sight, the time required to acquire a target is exponentially distributed.

5) Given continuous acquisition and target survival of attacks by other weapons, the time necessary to destroy a target is exponentially distributed.

6) For a given shooting weapon, *all* visible and invisible periods, acquisition times, and times-to-kill are mutually independent.

7) The entire line of sight, acquisition, and kill processes of different shooting weapons are mutually independent, except that a currently engaged target may be destroyed by another shooting weapon.

8) There is no extraneous damage (to other, nearby targets that are not under engagement, e.g.).

Parameters above depend upon types of both shooting and target weapons and possibly also on environment and range.

Given the level of detail of the ground combat portion of the VECTOR-2 model, Assumption 1) seems unjustifiably restrictive. There should be simple, reasonable alternatives.

Assumptions 6) and 7) are crucial in physical and mathematical terms. Physically, they imply that a shooting weapon is never aided in acquisition of one target by acquisition or destruction of another target and that shooting weapons on a given side act without coordination or synergistic effects, which may be viewed as a limitation of the model, albeit a nearly universal limitation. Furthermore, no shooting weapon may simultaneously engage more than one target, and acquisition of a target is not influenced by the fact that the target itself may be firing upon the shooting weapon. However, despite these somewhat unrealistic physical implications, the assumptions are

probably as complex as the current state of knowledge concerning stochastic attrition processes allows; the same assumptions, cf. [9,10,12], are present in the CEM and the IDAGAM I and Lulejian-I models.

Mathematically, as demonstrated and discussed in [7,11], Assumptions 6) and 7) determine the qualitative structure of the VECTOR-2 attrition process and, consequently, of the attrition equation [(6.1) below] chosen to approximate relevant expectations. Because of serial target selection, the stochastic attrition process engendered by Assumptions 1) - 8) above is not Markov, but is a semi-Markov process. The reader is referred to [4,5] for background and details concerning semi-Markov processes. Before we discuss the attrition process in more detail, we introduce necessary notation.

We describe attrition to weapons in a maneuver unit on the Red side during one (Level 1) time period; calculations for attrition to the Blue side are completely analogous. Let

$R_j(t)$ = number of type j weapons on Red side at time t,

$B_i(t)$ = number of type i weapons on Blue side at time t,

Δt = Level 1 time increment.

These numbers of weapons do not include weapons devoted to flank defense, weapons without ammunition, or (if a side is delaying) weapons at the next defensible position to the rear. Additional notation will be defined as necessary.

Assumptions 6) and 7) imply that expected losses of Red weapons of type j during the time interval $(t,t+\Delta t]$ are given approximately by

$$\Delta R_j(t) = R_j(t) - R_j(t+\Delta t) = [\sum_i a(i,j,B(t),R(t))B_i(t)]\ \Delta t\ . \tag{6.1}$$

The function a is the *attrition rate function* and depends on the current structures of the Blue and Red forces; its derivation is discussed below. Were the underlying attrition process Markov, derivation of (6.1) would be straightforward, based upon well-known properties of infinitesimal generators. In the semi-Markov case, more care and additional assumptions are required.

In terms of known and studied attrition models, (6.1) is not easy to classify. At first glance, the process appears, in the terminology of [11], to be of independent engagement initiation form, but only if one ignores (improperly) dependence of the attrition rate function on B(t) and R(t). One class of deterministic analogues of (6.1) is the family of "variable coefficient Lanchester models" extensively studied by S. Bonder and J. Taylor (cf., for example, [3,4,19]). Note that (6.1) can also be viewed, cf. [7,11], inaccurately but appealingly, in terms of interpretation and understanding, as arising from a *Markov* attrition process with infinitesimal generator A given by

$$(6.2) \qquad A[(x,y),(x;y_1\ldots,y_j-1,\ldots,y_N)] = \sum_i a(i,j,x,y)x_i$$

where $x \in R^M$ represents Blue forces, $y \in R^N$ represents Red forces (M and N are the numbers of Blue and Red weapon types, respectively).

Indeed, in some ways it seems to the reviewer that this latter interpretation of (6.1) is the most reasonable, even though it is not strictly compatible with Assumptions 1) - 8) above. In this interpretation, (6.1) is the standard approximation based on infinitesimal generators, and calculation of the attrition rate function as described below amounts to a careful, sophisticated, and reasonable derivation of the infinitesimal generator of the Markov attrition process.

A further complication is that the attrition rate function incorporates parameters (cf. (6.3), for example) that depend

on environmental factors and hence also on time; the resulting stochastic attrition process is, consequently, not temporally homogeneous. Nonhomogeneous semi-Markov processes are not well understood, while nonhomogeneous Markov processes, although not amenable to detailed computational treatment, are at least fairly well understood. Moreover, even in the nonhomogeneous Markov case, the interpretation and validity of (6.1) remain as previously discussed.

We now discuss derivation of the attrition rate function. The type i of shooting weapon will hereafter be fixed and dropped from the notation; all parameters below depend on it. Assume first that the shooting weapon employs parallel target acquisition; this is easier to handle than serial acquisition. Without loss of generality we suppose that opposition weapon types and priorities (for the given type of shooting weapon) coincide, which will simplify our exposition.

The data (model inputs) used to compute $a(j,B(t),R(t))$ in this case are for each target type k:

μ_k = exponential parameter for lengths of visible periods,

η_k = exponential parameter for lengths of invisible periods,

λ_k = exponential parameter for time to acquire given continuous visibility,

α_k = exponential parameter for time to kill given continuous acquisition and engagement.

The parameter of an exponential distribution is the inverse of its expectation, i.e., the rate of the associated Poisson process. These parameters may vary over time and may depend on the distance from the shooting weapon to the target.

The VECTOR-2 model incorporates the following expression, based on standard limiting results in renewal theory, for the attrition rate function:

$$(6.3)\quad a(j,B(t),R(t)) = \alpha_j\left[1-\left(1-\frac{\eta_j}{\mu_j+\eta_j}\,\frac{\lambda_j}{\mu_j+\lambda_j}\right)^{R_j(t)}\right]$$

$$\times\prod_{k=1}^{j-1}\left(1-\frac{\eta_k}{\mu_k+\eta_k}\,\frac{\lambda_k}{\mu_k+\lambda_k}\right)^{R_k(t)}.$$

The interpretation of (6.3) is both natural and reasonable:

1) For each k, $\eta_k/(\mu_k+\eta_k)$ is the probability that a given target of type k is visible;

2) For each k, $\lambda_k/(\mu_k+\lambda_k)$ is the probability of acquisition given visibility.

Therefore the indexed product on the right-hand side of (6.3) is the probability that no target of priority less than j is visible and acquired; the second factor is the probability that at least one type j target is visible and acquired; the product is [by Assumption 6)] the probability that the shooting weapon is currently engaging a target of type j. Multiplied by the rate of kill given engagement, this quantity becomes the appropriate attrition rate.

Observe that (6.3) ignores the possibility that a currently engaged target may be destroyed by some other shooting weapon. To the reviewer this seems a reasonable simplification; it is not, however, incorporated into attrition rate functions for shooting weapons with serial target acquisition, the case we describe next.

In the following discussion the type i of the serial acquisition shooting weapon is fixed throughout and suppressed from the notation; unless explicitly stated to the contrary, each parameter mentioned is a function of the type of shooting weapon. We assume, without loss of generality, that target type and target priority for opposition weapons coincide. Data

for the computation, provided by the user as inputs to the model, are:

$t_1 < \ldots < t_{N-1}$ = search cutoff times (cf. page 28 for an explanation),

μ_k = exponential parameter for visible periods of type k target weapons,

η_k = exponential parameter for invisible periods of type k target weapons,

λ_k = exponential parameter for time to acquire a type k target weapon, given continuous visibility,

α_k = exponential parameter for time to destroy a type k target weapon, given continuous engagement.

Factors which the attrition rate function $a(\cdot,\cdot,\cdot)$ explicitly represents are search cutoff times, loss of line of sight, and possible destruction of the target by another shooting weapon. As previously noted, the latter is ignored in parallel target acquisition; it would seem reasonable and more consistent to ignore it in this case as well.

Computation of the function a involves modeling the time-evolving state of a particular shooting weapon as a semi-Markov process $(Y_t)_{t \geq 0}$. For details concerning semi-Markov processes and Markov renewal processes, from which they arise, the reader is referred to [4,5]. Except for an important limit theorem, none of the results developed in [4,5] is needed in the VECTOR-2 model. The state space E of this semi-Markov process is given by

$$E = \{k_1, \ldots, k_N, \ell_1, \ldots, \ell_N\} ,$$

where

k_j = state of ultimately acquiring or actively engaging a type j target that will be destroyed by this engagement,

ℓ_j = state of acquiring or engaging a type j target that will not be destroyed by *this* engagement (because the line of sight is lost or the target is destroyed by another shooting weapon).

Let P be the transition matrix of the Markov chain embedded in (Y_t), i.e., the sequence of states entered by (Y_t) without reference to the time spent in each, and let $\overline{G}(x)$ be the expected length of a visit of the process (Y_t) to state x. By standard results for Markov chains, which are applicable at least if all types of opposition weapons are present, there exists a unique probability distribution ν on E such that

$$\nu = \nu P \ . \tag{6.4}$$

Then, by an important limit theorem for semi-Markov processes (cf. Theorems (10.4.3) and (10.5.22) of [5]),

$$\lim_{t\to\infty} P\{Y_t = x\} = \nu(x)\overline{G}(x)\Big[\sum_{y\in E} \nu(y)\overline{G}(y)\Big]^{-1} \tag{6.5}$$

for each $x \in E$. The attrition rate function is then taken to be

$$a(j,B(t),R(t)) = \nu(k_j)\Big[\sum_{y\in E} \nu(y)\overline{G}(y)\Big]^{-1} \ . \tag{6.6}$$

Roughly speaking, the right-hand side of (6.5) is for $x = k_j$ the probability of being in the state k_j, and $\overline{G}(k_j)$ is the expected time spent in k_j, at the end of which the target is actually destroyed, so the right-hand side of (6.6) is the *rate* at which the target is being destroyed. Note that the interpretation is based on infinitesimal behavior of the process, while (6.5) and (6.6) are based on limiting behavior.

There are certainly intellectual, and possibly practical, difficulties describing manifestly transient behavior of a process, which one wants to incorporate in differential models of combat, by means of limit theorems. A further difficulty

is that the transition parameters of the semi-Markov process (Y_t) depend on current force structures B(t) and R(t) that are themselves (random) functions of time; invocation of limit theorems that assume these to be constant is then inappropriate.

Because alternatives have not been developed and explored, the magnitude and implication of practical difficulties arising from use of (6.6) cannot be assessed. An alternative that more realistically represents transient aspects of the attrition rate function would be to take a(j,B(t),R(t)) to be the inverse of an appropriately defined expected first passage time to a (newly defined) state of "engaging a type j target that will be destroyed by this engagement."; cf. [17,18] for a more complete discussion of the relevant, but unapplied, mathematics.

The problem of computing the attrition rate function a is now reduced to computing the transition matrix P and the expected sojourn lengths $\overline{G}(x)$, $x \in E$. We shall deal with them in that order. First, for all $x \in E$ and each j

$$P(x,k_j) = q_j h_j$$
$$P(x,\ell_j) = q_j(1-h_j) \quad , \tag{6.7}$$

where

q_j = probability that the next target selected for engagement is of type j,

h_j = probability that a currently engaged target of type j is destroyed before loss of line of sight or destruction by another shooting weapon.

The numbers q_j, h_j must now be computed.

In the VECTOR-2 model, the probability q_j is computed by the equation

$$(6.8) \qquad q_j = d_j(t_{j-1}) \prod_{r=1}^{j-1} [1-H_r(t_{j-1}-t_{r-1})][1-d_r(t_{r-1})]$$

$$+ (1-d_j(t_{j-1})) \sum_{r=j-1}^{N}$$

$$\int_{t_r}^{t_{r+1}} \left(\prod_{\substack{s=0 \\ s\neq j-1}}^{r} [1-d_{s+1}(t_s)][1-H_{s+1}(x-t_s)] \right) H_j(dx-t_j) \ ,$$

where

$d_k(u)$ = probability that a target of type k has been and remains acquired u time units after initiation of search,

$H_k(\cdot)$ = distribution function of time to acquire a target of type k.

One must interpret (6.8) term by term:

1) The first factor in the first summand is the probability that a target of type j is acquired before t_{j-1} and hence eligible for selection at t_{j-1}; the product is the probability that no target with greater priority has previously been selected.

2) In the second summand, $1-d_j(t_{j-1})$ is the probability that a given target of type j is not acquired at t_{j-1} and hence not immediately available for selection then. The other factor represents the probability that no other target (of any priority) is selected first; r+1 is the type of the lowest priority target for which search is actually initiated.

The remaining calculations are not hard to describe; to compute $H_k(\cdot)$ the model uses the equation

$$(6.9) \qquad H_k(u) = 1 - \exp\left(- \frac{\lambda_k \eta_k}{\eta_k+\mu_k} R_k(t)u \right) ,$$

where t in $R_k(t)$ is not a dummy variable but the initial point of the time step under consideration. Since $\eta_k/(\eta_k+\mu_k)$ is the

probability that a particular type k target is visible and λ_k the rate of acquisition given continuous visibility, (6.9) is reasonable within the context of the other computations.

Finally, to compute $d_k(u)$ the model employs the equation

$$(6.10)\quad d_k(u) = 1-\left[1 - \frac{r_k}{\mu_k+\rho_k-r_k}\left(e^{-r_k u} - e^{-(\mu_k+\rho_k)u}\right)\right]^{R_k(t)} ,$$

where

$$r_k = \frac{\lambda_k \eta_k}{\eta_k+\mu_k}$$

is interpreted as above and where $\rho_k = \rho_k(B(t))$ is the rate at which *other* shooting weapons are destroying a given target of type k (details of the computation of which may be found in [14]). As previously noted, we believe it would be a reasonable and consistent simplification to take $\rho_k = 0$.

To compute the destruction probability h_j the equation used is

$$(6.11)\qquad h_j = \frac{\alpha_j}{\alpha_j + \mu_j + \rho_j} ,$$

which seems entirely reasonable: once begun, the engagement ends in destruction of the target at rate α_j, in loss of line of sight at rate μ_j and in destruction of the target by another shooting weapon at rate ρ_j.

To complete the calculations necessary to use (6.6), it remains to compute $\overline{G}(x)$ for each $x \in E$. To do this first write

$$\overline{G}(k_j) = E_{k_j}[T] + E_{k_j}[W]$$
$$\overline{G}(\ell_j) = E_{\ell_j}[T] + E_{\ell_j}[W] ,$$

where

T = time spent in acquisition and selection,

W = time spent in firing,

and where $E_x[Z] = E[Z|Y_0 = x]$. From (6.8) it then follows immediately that

$$(6.12)\quad E_{k_j}[T] = E_{\ell_j}[T]$$

$$= t_{j-1}d_j(t_{j-1}) \prod_{r=1}^{j-1} [1-H_r(t_{j-1}-t_r)][1-d_r(t_{r-1})]$$

$$+ (1-d_j(t_{j-1}))$$

$$\times \sum_{r=j-1}^{N-1} \int_{(t_r,t_{r-1}]} x \left(\prod_{\substack{s=0 \\ s\neq j-1}}^{r} \left[1-d_{s+1}(t_s)\right] \times \left[1-H_{s+1}(x-t_s)\right]\right) H_j(dx-t_{j-1}) .$$

Finally,

$$(6.13)\quad E_{k_j}[W] = E_{\ell_j}[W]$$

$$= \frac{1}{\alpha_j + \mu_j + \rho_j} ,$$

which follows from (6.11). The expected length of an engagement is independent of the outcome, as (6.13) states. This completes derivation of the quantities needed to employ the basic attrition equation (6.1). The report does not specify how computational difficulties involving numerical aspects of (6.3) - (6.13) are surmounted in the model.

In Appendix E of [14], several detailed methods for calculation of kill rates α_j corresponding to different firing doctrines are presented, namely--

1) Single-shot with Markov fire adjustment, with hit necessary to destroy target,
2) Single-shot with Markov fire adjustment, with possibility that a missed shot destroys the target by killing the associated personnel,
3) Burst fire with time between bursts but no fire adjustment,
4) Area-lethality mechanism.

The reader is referred to [14] and also to [3] for details of these computations.

We conclude this Section with a description of the equations used to compute attrition resulting from support fire; that is, fire by artillery and mortars or by attack helicopters functioning in a support fire role against artillery batteries, air defense sites, and maneuver units. Factors that determine computed attrition are numbers and types of firing weapons, type of ordnance and delivery technique, target type and activity, environment, type of sensor reporting the target, and accuracy and timeliness of the report. Two qualitatively different ordnances are represented; individually targeted (hit-to-kill) ordnance and area-targeted (area effects) ordnance.

For individually targeted ordnance, the attrition equation is

$$\Delta t_j = [f_j \sum_{\ell} k(j,\ell)p(j,\ell)]M \ , \tag{6.14}$$

where

j = target type,
t_j = number of type j targets vulnerable,
Δt_j = attrition to targets of type j,
ℓ = posture class of targets,
$p(j,\ell)$ = fraction of type j targets that are in posture class ℓ,
$k(j,\ell)$ = probability that one unit of expended ordnance directed at a target of type j in posture class ℓ destroys the target,

f_j = probability that a type j target is chosen to be attacked,

M = number of ordnance units expended.

It appears that a computation of the form (6.14) is performed for each weapon functioning in a support fire role and that effects are then summed. It is generally the case that

$$f_j = \frac{t_j}{\sum_k t_k} ,$$

whereupon (6.14) is of independent engagement initiation form. Applicability of (6.14) to individually targeted ordnance is then possibly inappropriate, especially in terms of underlying assumptions; the reader is referred to [7,11] for further details.

For area-targeted ordnance it is assumed that each target (a maneuver unit, say) is partitioned into one or more equivalent subtargets, no two of which can be damaged by a single attack. Attacks against the entire target are allocated uniformly among the subtargets. To compute, for a given subtarget, the attrition to target elements comprising that subtarget, the VECTOR-2 model uses the equation

$$\Delta t_j = t_j[1 - (1-d_j)^N] , \tag{6.15}$$

where

j = type of target element,

t_j = number of target elements of type j in the subtarget,

Δt_j = attrition to type j target elements in the subtarget,

d_j = fractional damage to one target element of type j in one attack on entire subtarget,

N = number of attacks on subtarget.

As appropriate, the number of attacks, N, is the number of artillery volleys fired at a given subtarget or the number of aircraft (in the CAS mission) attacking the subtarget. To

compute the fractional damage d_j resulting to one target element from one attack, the VECTOR-2 model uses the equation

$$(6.16) \qquad d_j = \sum_{k=1}^{M} (-1)^{k-1} \frac{\dfrac{c_j^2 + s^2}{q^2}}{\dfrac{c_j^2 + s^2}{q^2} + k} \frac{(M)_k}{k!} \left(\frac{c_j^2}{c_j^2 + s^2} \right)^k ,$$

where

M = number of firing "patterns" constituting one attack,

q = circular error probability of center of pattern around subtarget,

s = subtarget size parameter,

c_j = lethality radius of one pattern against target element of type j,

and where $(M)_k = M(M-1) \cdots (M-k+1)$.

Equation (6.16) is based upon target elements being uniformly distributed over the area comprising the subtarget and on normally distributed delivery errors; cf. [14] for details.

For small values of the d_j, equation (6.15) becomes

$$\Delta t_j \sim d_j t_j N ,$$

which is an attrition equation of proportional engagement initiation form, whose applicability to "area-targeted" ordnance is uncertain; cf. [7,11].

The reader should note that only resource groups in the first band of zones across the FEBA are vulnerable to artillery; in particular, usable air bases are not vulnerable to artillery.

7. ATTRITION IN AIR COMBAT

In this Section we describe the methodology employed in VECTOR-2 for computing losses in air-to-air interactions, ground-to-air interactions, and air-to-ground interactions other than those involving maneuver units. Compared to that used for assessing the effects of ground combat interactions, the air combat methodology is both simple and simplistic, possessing neither the overwhelming level of detail nor the degree of sophistication of the former. The air-to-air combat model is structured on the basis of air groups, which are constructed and have missions assigned to them by the tactical decision rules discussed in Section 5. Construction of air groups and assignment of missions seem to be done on the basis of sectors; however, time evolution of the location and status of each group is represented individually. Air-to-air combat involving a flight of attacking aircraft and associated escorts will occur if enemy target acquisition resources acquire the attacking air group, if a flight of interceptors is allocated (as determined by tactical decision rules) to attempt an intercept, and if the attempted intercept is successful. It is not clear how success or failure of an attempted intercept is determined.

Let us now consider an engagement involving attacking aircraft, escorts, and interceptors under the assumption of a successful intercept. Let

A_i = number of attacking aircraft of type i,
E_j = number of escort aircraft of type j,
I_k = number of interceptors of type k,

denote the respective numbers of aircraft involved. Once the engagement is initiated, movement of aircraft involved ceases until engagement effects are calculated, which is a reasonable procedure. An air-to-air engagement consists of one or more long-range phases involving stand-off weapons followed by a short-range duel phase. During each phase targets are selected randomly from those acquired and simultaneous engagement of more than one target is permissible. After each phase of the engagement, either side can disengage (using tactical decision rules). For disengaging penetrators, the mission is considered aborted and aircraft turned back toward their bases, apparently unable to attempt to resume their original mission. If interceptors disengage, penetrators continue toward their targets. Engagements can occur involving outbound as well as inbound attacking aircraft but, unreasonably, appear to be treated identically.

A single form of attrition equation is used to compute aircraft losses to both sides in all phases of air-to-air combat. While this is consistent, it also seems an undesirable and unnecessary oversimplification. In particular, it fails to account for asymmetry of the objectives of interceptors and penetrators, which is represented, for example, in the barrier penetration attrition model described in [1]. According to the VECTOR-2 equation, the number of attacking aircraft of type i destroyed in a given phase of engagement, ΔA_i, is given by

$$\Delta A_i = \frac{A_i}{A + E} \sum_k a(i,k) \min\{A_i, m_k\} I_k , \tag{7.1}$$

where

$a(i,k)$ = "attrition caused by one interceptor of type k that engages only attacking aircraft of type i",

m_k = maximum number of aircraft that an interceptor of type k can engage simultaneously,

and where

$$A = \sum_i A_i$$

and

$$E = \sum_j E_j$$

are the total numbers of attacking aircraft and escorts, respectively. The parameters a(i,k) and (possibly) m_k depend upon the phase of the duel. Equation (7.1) is, for large numbers of targets, of independent engagement initiation form (in the taxonomy of [11]) for if $A_i \geq m_k$ for all k, then (7.1) becomes

$$\Delta A_i = \frac{A_i}{A + E} \sum_k a(i,k) m_k I_k \; ; \tag{7.2}$$

the factor $A_i/(A+E)$ represents a uniform fire allocation, cf. [7]. The a(i,k) are only vaguely defined in [14], in the words quoted following (7.1), which make them seem kill potentials of the usual undefinable and uncomputable sort. Observe also that (7.1) allows multiple engagements only against attacking aircraft of the same type, an unwarranted and restrictive assumption.

Use of an independent engagement initiation attrition equation to represent air-to-air combat involving large numbers of targets seems reasonable even though the fire allocation in (7.1) represents fairly precise coordination and information transfer by the defense. When engagement capabilities of interceptors become unlimited (i.e., $m_k \to \infty$ for all k), then (7.1) becomes an attrition equation with proportional engagement initiation:

$$\Delta A_i = A_i \left(\frac{A_i}{A+E} \right) \sum_k a(i,k) I_k \; , \tag{7.3}$$

which also is plausible. Note that (7.3) obtains for small numbers of attacking aircraft, in which case engagement opportunities become constrained by the numbers of available targets.

In [13] the reviewer derives from explicit physical and probabilistic assumptions an attrition equation to which (7.1), (7.2), and (7.3) can be regarded as linear approximations; the interested reader is referred there for further details. Another derivation appears in [6].

For completeness we observe that losses ΔE_j of escorts of type j are given by

$$\Delta E_j = \frac{E_j}{A + E} \sum_k a'(j,k) \min\{E_j, m_k\} I_k \ , \tag{7.4}$$

where a'(j,k) is the attrition caused by one interceptor of type k that exclusively engages escorts of type j. Similarly, losses ΔI_k of interceptors of type k are given by

$$\Delta I_k = \frac{I_k}{I} \left[\sum_i \tilde{a}(k,i) \min\{I_k, \tilde{m}_i\} A_i + \sum_j \hat{a}(k,j) \min\{I_k, \hat{m}_j\} E_j \right] , \tag{7.5}$$

where I is the total number of interceptors, the $\tilde{a}(k,i)$ and $\hat{a}(k,j)$ are attrition potentials, and the $\tilde{m}_i$ and $\hat{m}_j$ are maximum numbers of simultaneous engagements.

While (7.1), (7.4), and (7.5) are not individually unreasonable, together they have several deficiences. First and possibly most important is that they ignore differing objectives of attacking aircraft, escorts, and penetrators, unless one represents them by the attrition potentials. Second, consideration of time sequencing of events seems to be ignored; in particular it seems that interceptors would often encounter escorts before attackers, so that only interceptors surviving interactions with escorts should be allowed to engage attacking aircraft. This coarse treatment of air-to-air combat is a disappointment.

Next we consider losses to ground weapon systems of aircraft on overflight (i.e., aircraft whose targets are not immediately defended by the ground weapon systems). This portion of the VECTOR-2 model is even more simplistic than the representation of air-to-air combat, but still seems reasonable: losses on overflight probably do constitute a nonnegligible but not significant component of aircraft attrition.

Aircraft on overflight are fired upon by air defense weapons provided that--

1) the aircraft be acquired by the air defense weapons;
2) the air defense weapons not be engaging targets of higher priority (aircraft attacking either the weapons themselves or a target they defend);
3) the aircraft not be in an intercept corridor (in which interceptor aircraft are also present);
4) ordnance be available.

Inputs to the attrition calculation are firing rates for different types of air defense weapons; other quantities appearing below are internally computed.

Below, the indices "g" and "h" denote groups of attacking aircraft, while "j" and "k" denote types of attacking aircraft. Let

$A(g,j)$ = number of type j aircraft in group g,

D_i = number of type i air defense sites to which aircraft are vulnerable,

$\beta(j,i)$ = *rate* at which one type i air defense site destroys aircraft of type j, given continuous engagement,

Δt = time interval under consideration (Level 1).

The $A(g,j)$, D_i, and the time interval Δt are calculated within the model, the first two from explicit positional data. Losses of attacking aircraft of type j in group g, $\Delta A(g,j)$, are then given by

$$\Delta A(g,j) = \left[\sum_i \beta(j,i)e(g,j,i)D_i\right]\Delta t \ , \tag{7.6}$$

where the e(g,j,i) are allocation factors computed according to the equation

$$(7.7) \quad e(g,j,i) = \frac{p(g,j,i)A(g,j)}{\sum_{h,k} p(h,k,i)A(h,k)} \left[1- \prod_{h,k} (1-p(h,k,i))^{A(h,k)}\right] .$$

In Equation (7.7), p(g,j,i) is the proportion of time an aircraft of type j in group g is in range of and acquired by a (representative) air defense side of type i; it is computed within the model from data concerning aircraft flight paths and "locations" of air defense sites. Observe that with i fixed

$$\sum_{g,j} e(g,j,i) = \left[1- \prod_{h,k} (1-p(h,k,i))^{A(h,k)}\right]$$

so that (7.6) and (7.7) account for weapons with no targets in range and acquired. Implicit in (7.7) is the uniform fire allocation represented by the first factor on the right-hand side. This assumption is not unreasonable; the further complications involved in more detailed schemes seem unnecessary.

Finally we discuss aircraft-related combat effects in the target area; specifically, since air-to-ground losses are treated in Section 6 on attrition in ground combat, we described computation of losses of attacking aircraft inflicted by air defense sites in the immediate vicinity of the target. The target may be an air defense site. The following steps constitute this interaction.

1) An aircraft group arriving in the vicinity of its preassigned target must reacquire the target in order to attack it. If reacquisition is unsuccessful, an attempt may be made to acquire a specified secondary target (which seemingly must be nearby). If no target is acquired, the attack is aborted.
2) If a target is acquired, all aircraft (except escorts) in the group attack it. Escorts are not vulnerable to target area defenses.

3) Attacking aircraft make one or more passes over a target. At the start of each pass, air defense weapons fire upon the aircraft, causing attrition that is calculated in equation (7.8) below. The attacking side may abort the attack at the end of any pass if losses are excessive. Each successful pass by one aircraft results in delivery of a specified type and amount of ordnance, the effects of which are described in Section 6.

The following equation is used in VECTOR-2 to compute losses of aircraft to target area defenses on *one pass* over the target. Let

$\tilde{A}_j$ = number of aircraft of type j that survive to make the pass over the target,

D_i = number of type i air defense sites defending the target,

$t(j,i)$ = length of time in one pass that an aircraft of type j is acquired by an air defense site of type i,

$\gamma(j,i)$ = *rate* of kill given continuous acquisition and engagement of type j aircraft by type i air defense site.

The $\gamma(j,i)$ are inputs to the model; the remaining quantities above are calculated and dynamically updated within the VECTOR-2 model. Losses of type j aircraft on one pass, $\Delta\tilde{A}_j$, are then given by the equation

$$\Delta\tilde{A}_j = \frac{\tilde{A}_j}{\sum_k \tilde{A}_k} \sum_i \gamma(j,i)t(j,i)D_i \ . \tag{7.8}$$

As noted earlier in this Section and in [11], this attrition equation is of independent engagement initiation form and, consequently, seems reasonable in this context.

8. MOVEMENT OF GROUND UNITS AND AIRCRAFT

In terms of representation of FEBA location and movement, VECTOR-2 is clearly superior to the other models. Unlike the IDAGAM I, Lulejian-I, and CONAF Evaluation models, the VECTOR-2 model does not contain a FEBA movement computation *per se*; in particular, FEBA movement is *not* computed using functions based on historical data whose argument is a force ratio. Rather, movement of ground force maneuver units is represented explicitly and computed positions are updated each Level 1 time period. FEBA position is simply the line of farthest current advance of front line maneuver units and changes over time as maneuver unit locations vary. Therefore, FEBA position is determined internally from explicit geographical locations of resources, rather than--as in the other models--by an externally and artificially imposed function of force ratios.

To avoid ambiguities in possible definition of the FEBA and because of an inherent difficulty in representing breakthroughs and encirclements (which are physically identical but interchange the roles of attacker and defender), the latter are not allowed. Even with this restriction, VECTOR-2 remains much more realistic and detailed than the three other models.

To obtain the current location of the FEBA (for each side--the sides are generally separated by a positive distance as described in Section 3), it suffices to observe the positions of front line maneuver units. Evolution of these positions, in turn, occurs subject to the following assumptions:

1) Reorganizational movement occurs instantaneously at the start of each Level 6 time period if the distances

involved are sufficiently short; for more extended movements a period of unavailability is represented.

2) Tactical movement of maneuver units is computed in terms of the coordinate geography and occurs in straight lines except when (in movement of a reserve unit) it would be necessary to cross the FEBA.

3) Movement occurs at input-prescribed rates that are functions of weather, terrain, and possible presence of a minefield.

4) During close combat engagements, effects of range and movement are represented by variations in parameters defining different line of sight processes. This dependence is not described explicitly in [14] but some aspects of it are treated in [3].

FEBA movement is computed and available as an output for each front line combat arena. For each side, FEBA movement is by definition the physical movement of the front line force in that arena.

Movement of ground resource groups other than maneuver units is not included in the model; such resource groups are assumed to possess sufficient capability to maintain their prescribed distances from the FEBA (cf. Section 3).

Location and movement of air groups, namely attack and escort groups on the attacking side and interceptor groups on the defending side, is--like maneuver unit location and movement--represented explicitly in terms of coordinate battlefield geography and updated each Level 1 time period. The following assumptions are in force during these calculations:

1) Each air group is represented by a single coordinate position.

2) Flight paths consist of straight line segments transversed at input-specified speeds. Flight paths are constructed and selected by tactical decision rules that assign aircraft missions.

3) An air-to-air or air-to-ground engagement stops progress along a flight path for the duration of the engagement.

Flights of attack helicopters functioning in the support fire role (rather than as maneuver unit weapons) are treated in the same way.

A significant problem in theater level combat modeling is determination of *engagement eligibility*: what targets are vulnerable to what shooting weapons? Some models, because of imprecise rules for this determination, seem to calculate unreasonably high losses. VECTOR-2, on the other hand, can deal easily and realistically for the most part with engagement eligibility by calculating it from explicit positional data. But when some relevant locations, such as those of air defense sites, are notional, whereas other locations, such as those of overflying aircraft, are explicit, the methodology encounters the same problems that plague other models.

9. INTELLIGENCE AND TARGET ACQUISITION

The important physical phenomena of intelligence and target acquisition are represented in more detail and more realistically in VECTOR-2 than in the CONAF Evaluation, IDAGAM I, and Lulejian-I models. In the latter models, target acquisition is generally represented, if at all, only in terms of ill-defined probabilities of detection. Target intelligence is usually nonexistent and weather intelligence is not present since weather itself is not represented. Target intelligence and acquisition for weapons in maneuver units have been treated in detail in Section 6 and will not be discussed further here. After some preliminary remarks, however, we will discuss target acquisition for firing weapons in support fire roles.

Intelligence represented in VECTOR-2 is of four kinds:

1) Weather intelligence as user-input five day weather forecasts for each side;
2) Target intelligence for maneuver units, air defense weapons, and aircraft, as previously described in Sections 6 and 7;
3) Target intelligence for allocation of support fire, as expected numbers of acquisitions by observation resources, expected location errors, and expected times since acquisition;
4) Enemy order-of-battle intelligence, as Bayesian estimates of numbers of ground targets by type and zone (cf. Section 3) and numbers of aircraft by sector.

The third and fourth categories will be discussed in more detail below.

Intelligence concerning terrain and enemy posture are not represented in the model; both are assumed to be perfect. Alternative assumptions may be possible through particular sets of tactical decision rules.

Concerning target acquisition for support fire (by artillery, helicopters or aircraft), the fourteen types of observation resources are partitioned into two classes:

1) Observation resources that do not adjust fire but simply report, subject to error, the location of an acquired target and then resume search efforts;
2) Observation resources that adjust fire and attempt to maintain acquisition, with negligible error, until the fire is delivered.

The following enemy resource groups are vulnerable to acquisition:

1) Maneuver units
2) Artillery
3) Air defense sites
4) Command posts
5) Logistic targets
6) Air bases
7) Enemy target acquisition resources
8) Aircraft in flight
9) Two user-specified classes of resource groups.

Acquisition of a target requires coverage of the target (it must be in range and in the field of view of the observation resource) and existence of a line of sight. Acquisition information is retained in the model until support fire is allocated to the target or the target becomes invisible, or until the information becomes too old to be reliable (the target can move). It is further assumed that there is neither acquisition of false targets, nor misidentification of targets, nor cueing of one observation resource by another.

Subject to these assumptions, the probability that a particular observation resource acquires a particular target in t time units or less is given by

$$p(t) = 1 - \left[1 - p_C p_L (1-e^{-\lambda t})\right]^S , \tag{9.1}$$

where

$p(t)$ = probability of acquisition in $[0,t]$,

S = number of sensors comprising observation resource,

p_C = probability of coverage for each sensor,

p_L = probability of existence of line of sight for each sensor,

λ = acquisition rate given coverage and existence of line of sight.

The latter three quantities are computed within the model from data concerning relevant positions, terrain, and weather; details are not given in [14].

Searches are conducted by different sensors in a user-prescribed order; previously acquired targets are not reacquired. Hence each sensor decreases the set of unacquired targets by acquiring targets undetected by prior searches; in this way [14, p. D-2] acquisition data are "reported...by the most preferred sensor system able to achieve detection."

Moreover, target acquisition entails processing time and possible communication delays, so that it is possible that a sensor system may become saturated and unable to make further detections or reports.

Outputs of this process, namely expected numbers of acquisitions of targets against which support fire has not previously been allocated, together with location errors and times since acquisition, become inputs to the equations that calculate attrition effects of support fire.

The Bayesian scheme for updating enemy order-of-battle intelligence is, for a single type of target, of the form

$$\hat{n}_c = p(t)n_c + (1-\hat{p}(t))\hat{n}_p \ , \tag{9.2}$$

where t is the length of the time interval under consideration, $p(t)$ is given by (9.1), and

$\hat{n}_c$ = estimated current number of targets,

n_c = actual current number of targets,

$\hat{p}(t)$ = estimated acquisition probability,

$\hat{n}_p$ = previous estimate of number of targets.

This equation is reasonably realistic and adequately simple.

10. COMPUTER-RELATED ASPECTS

In this Section we briefly describe some computer and data requirements and features of the VECTOR-2 model. Our treatment is neither complete nor intense; the interested reader is referred to [15] for details.

Requirements of VECTOR-2 for input data, computer core, and computer running time are large, often exceeding significantly (by a factor of 2 or more) analogous requirements of the CONAF Evaluation, IDAGAM I, and Lulejian-I models. To some extent severity of these requirements is mitigated by ancillary computer programs such as the data preprocessor described below; however, even if all the reductions discussed below were effected, we believe that VECTOR-2 would continue to require more inputs, more core, and more running time than the other three models.

Since it is essentially an immutable characteristic of the model, the core requirement of

1) 150,000 32 bit words for computations involving each sector, and
2) 50,000 32 bit words for theater-level calculations

is difficult to change. The computer program is structured, however, so that sectors are computationally dependent only through theater-level calculations, which makes simultaneous consideration of 2 or more sectors never necessary. Some reduction in core requirements might be achieved by decreasing the numbers of weapon types or combat arenas, but is not likely to be significant.

Input data requirements are approximately:

1) Data concerning theater-level performance, tactics, and terrain variables (about 9500 values);
2) Data concerning sector-level performance, tactics, and terrain variables (about 21500 values);
3) Initial data, force arrivals, and weather information arrivals (about 1500 values);
4) Run control parameters (about 50 values).

One also must regard the 34 tactical decision rules as required inputs; the user wishing to derive these latter inputs faces a substantial burden in terms of analysis and programming (the tactical decision rules must be FORTRAN subroutines compatible with the main program). However, VECTOR-2 is based on physically measurable performance data; it does not entail the same amount of data preparation (as opposed to number of data values) as do the other models; for the latter, inputs such as "kill potentials" or "kill probabilities" must be calculated (basically by hand) from more primitive, often unspecified, data. Often, there is no suggested, let alone justified, method of making calculations.

Furthermore, the physically measurable inputs to VECTOR-2 can be prepared automatically by use of a *Data Preprocessor*, which extracts these inputs from a data base maintained by OSD. The Data Preprocessor is able to prepare inputs representing weapons system performance, order of battle, logistics, and communications. The user still must manually prepare data related to environment and scenario and to force numbers and arrivals, and also make any changes needed in automatically prepared data.

We emphasize that the greater input data requirement of VECTOR-2 is in itself neither an advantage nor a disadvantage of the model; it is a *difference* between VECTOR-2 and the other models. Some of the comments above depict it as a disadvantage; However, it is an advantage to the extent that more--and

physically definable--phenomena affect the results of the model. Certainly the input requirement is consistent with the level of detail of the model and perhaps is best judged (if judgment is the objective) in that larger context: if the level of detail is deemed appropriate, the input requirement must be accepted; if the level of detail is unnecessarily high, so is the input requirement.

The computer program for the VECTOR-2 model is very long: 12,000 FORTRAN statements comprising 20,000 lines of code. In order to be more efficient, the main program, including the tactical decision rule subroutines, accepts binary inputs and produces (for every run) a full set of binary outputs. The main program is complemented by a binary data formatter that produces necessary binary inputs and a *Postprocessor* that prepares user-selected output tables from the full set of binary outputs. An advantage of this structure is that, after examination of selected outputs, further outputs of interest can be obtained with relatively little effort; other models might have to be run again.

The available outputs are standard, as the following list demonstrates:

1) Summary outputs provided once per day: resource inventories at FEBA, theater losses, and FEBA position.
2) Theater-level outputs provided once per day: resource strengths by sector, resource losses by type and sector, loss attributions, FEBA positions and movement, intelligence, resource arrivals and allocations to sectors, aircraft and shelter inventories by sector, aircraft allocations to missions.
3) Sector-level outputs provided once per day: weather, map of zones, resource groups by zone and composition, maneuver unit hierarchy, supply distribution and consumption, front line combat effects.
4) Sector-level outputs provided several times per day: resource losses and loss attributions, maneuver unit

losses, target acquisition and support fire allocation, air groups flown, suppression and damage of air defense weapons.

5) Combat arena level outputs provided several times per day: status, inventories, and losses of front line units, attribution of maneuver unit losses, log of events.

A possibly significant property (as yet not fully documented) of the VECTOR-2 model is its running time requirement. The level of detail and shortness of suggested Level 1 time periods (see page 7) imply running times orders of magnitude greater than those of the comparable models. Such running times are not, *a priori*, a deficiency; they should be viewed as part of the price the user must pay for the model's detail in both physical and temporal senses. The report [14] includes no data concerning running times for campaigns on the order of 90 days; it also states that (at the time [14] was prepared) no such runs had been accomplished. If, for a campaign of prescribed duration, running time based on the level sizes suggested in Section 2 is unacceptably high, time steps could be increased. For example, a reduction of 90 percent in running time has been reported to result from the following changes:

CHANGE	FROM	TO
Level 1	30 seconds	15 minutes
Level 2	3 minutes	1 hour
Level 3	15 minutes	2 hours
Level 4	1 hour	2 hours

These changes reduce the daily number of Level 1 evaluations per combat arena from 2880 to 96. It would be worthwhile to perform a careful analysis of the resultant changes in output values.

An extremely useful computer tool is the *Program Change Monitor* described in detail in [15]. This program is designed to aid the user in the generally difficult task of changing the program (e.g., deleting certain variables) and seems to be

an excellent idea. For changing dimensions of arrays, however, alternative methods are available; for example, in IDAGAM I some dimensions (e.g., numbers of weapon types) are themselves inputs to the model (subject to upper bounds of course).

A final, worthwhile feature of the model is its "restart" capability. At the end of each run, final values of evolving state variables are written onto a magnetic tape. Should it then be necessary or desirable to continue that run this tape may be used as input and the previous run is resumed where it left off. There is no need to begin anew.

The reviewer has not analyzed the VECTOR-2 computer program and cannot vouch for consistency with published documentation or numerical aspects.

11. SUMMARY

Because of its level of detail, explicit coordinate geography, and extremely fine time divisions, the VECTOR-2 model is *potentially* a qualitative as well as quantitative advance over other existing models (CEM, IDAGAM I, Lulejian-I, and VECTOR-1). The increased fidelity of representation of space, time, attrition, and movement is impressive, but the word "potentially" is not to be ignored. In particular, the (undoubtedly significant) potential of the model may not be realized for any of the following reasons:

1) The model may not run in an acceptably short length of time when used at the full extent of its capabilities to represent geography and--more importantly--time. With time steps of the same order as those of comparable models, VECTOR-2 becomes less distinguishable from them. To our knowledge it is not yet known whether VECTOR-2 requires impractically large running times.

2) Even if its geographical and temporal representations can be fully exploited, VECTOR-2 may not produce discriminations that will be believed by model users but that could not also be obtained with less complicated, faster, and cheaper models. The prevalent reaction to theater-level models is extreme skepticism; the models are regarded as capable (at best) of making gross qualitative distinctions among force structures, weapon characteristics, and tactics. It seems agreed that no credence should be accorded to actual values of outputs or pairwise comparisons based on nearly identical outputs. Given this (reasonable) attitude of distrust, it

it may be that the believable and usable results obtained using VECTOR-2 might more cheaply, easily, and quickly be obtained using another model.

On the other hand, the VECTOR-2 model may be judged on essentially an intellectual basis as sufficiently more detailed and realistic than the comparable models. As such, (finer) distinctions obtained using it will be believed relative to those of the comparable models. We feel that quantitative results of theater-level models will never be credible in an absolute sense, but VECTOR-2 eventually may be viewed as producing more believable and finer qualitative distinctions. Should this happen, VECTOR-2 will be an order-of-magnitude improvement over the other models in practice as well as potential.

In terms of more specific conclusions, we list below the distinguishing and significant qualities of the VECTOR-2 model. As is often the case with a model, the same qualities are at once its greatest strengths and greatest liabilities. Flexibility, detail and realism, complexity, mathematical sophistication, and so on are strengths only if they are understood, respected, and not abused. Unfortunately, the greater the potential strength, the greater the opportunity for misunderstanding or misapplication, the latter being more user faults than model faults.

The distinguishing and significant qualities of the VECTOR-2 model are as follows:

1) The detailed representation of space, time, movement, and attrition constitutes the greatest single strength of the VECTOR-2 model. There is greater fidelity to the physical reality of combat, and also fewer arbitrary aggregations, fewer arbitrarily imposed sequences of events, and more reliance on definable and measurable inputs. In particular, computation of engagement eligibility and--especially--FEBA movement from

internal and explicit geographical data, represent significant advances over the methods used in the comparable models. Moreover, the VECTOR-2 structure allows for actual rather than artificial attribution of losses, permits dynamic representation of the effects of support fire, and eliminates the artificial front-to-flank ratios present in the other models.

2) The VECTOR-2 model represents (sometimes rather simplistically) phenomena such as target acquisition, communication delays, and imperfect knowledge of opposition forces and intentions that are treated either not at all or very vaguely in other models. While the treatment may not be the best, at least the user is made more aware of limitations of the model and of the arbitrary but necessary assumptions involved.

3) The tactical decision rules in VECTOR-2 permit the user essentially unlimited flexibility and freedom in representing decision making processes relevant to the combat, provided that he or she has the resources to exploit the tactical decision rule structure. Adaptive resource allocation schemes that are impossible in some of the other models are quite feasible with VECTOR-2. Furthermore, even those models (notably CEM) with adaptive decision making structures do not allow the structure itself to be changed from run to run; such changes are possible using VECTOR-2 if the user can produce alternative tactical decision rules. Finally, even if only the demonstration set of tactical decision is used, the structure as a whole serves to isolate and correlate decision making processes in the model; as a consequence, these processes become easier to understand both individually and collectively than in other models.

4) The VECTOR-2 level of detail, in addition to being significantly greater in many respects than those of other models, also generally is consistent internally. Some processes, such as target acquisition and communication delays, are given justifiably simplistic treatments. The exception

(as also in VECTOR-1) is the air combat portion of the model. Although movement is represented in extreme detail, engagements among aircraft and fire of air defense weapons at aircraft are not.

5) The extreme detail and complexity of the model are not without drawbacks in addition to that of running time. There is the difficulty of understanding the model sufficiently well to use it intelligently. Also, while the detail of the model means that none of the arbitrary choices necessary to construct the model has a significant effect in itself, there are many more choices to be made. While the net effect is not certain, it seems that the choices will tend to balance out so that the overall effect of arbitrary choices will be less than in the comparable models.

6) In order to better understand the VECTOR-2 model in both absolute and relative (to other models) terms, it may at some time be useful to consider the following questions.

a) Does a simplified attrition structure materially alter the outputs of the model?

b) What is the effect of increases in the sizes of lower level time steps?

c) Does a more complicated air model that is more consistent with the ground portion of the model (and includes, for example, a more realistic variety of aircraft missions) change the behavior of the model as a whole?

d) Can a simplified, fast-running version of the model, which ignores, for example, weather or communication delays and imperfect target acquisition, or combines acquisition and line of sight processes, be devised? How and to what extent does the simplified version differ from the fully detailed version of the model and from CEM, IDAGAM I, and Lulejian-I?

7) Except that it does not describe the demonstration package of tactical decision rules, the model documentation [14,15, 16] is excellent in terms of completeness and clarity. It has made this review feasible.

If there is a negative conclusion to be drawn, it is that VECTOR-2 is not really a theater-level model, although it may be the best corps-level model. In support of this, one may cite the oversimplified air model and the data and running time requirements entailed by use of VECTOR-2 in theater-level analyses analyses. As a purely mathematical structure, the VECTOR-2 model is without question more flexible, general, and realistic than the CONAF Evaluation, IDAGAM I, and Lulejian-I models. Whether practical use can be made of these advantages, though, is uncertain.

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REVIEW AND CRITIQUE OF THE VECTOR-2 COMBAT MODEL

Alan F. Karr

December 1977

INSTITUTE FOR DEFENSE ANALYSES
PROGRAM ANALYSIS DIVISION

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is a review and critique of the VECTOR-2 theater-level simulation model of conventional ground-air combat. Aspects of the model analyzed are organization and general structure, environment, resources and logistics, command, control and communication processes, attrition in ground combat, attrition in air-combat, movement of ground units and aircraft, intelligence and target acquisition, and computer-related aspects of the model. The paper emphasizes

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identification and understanding of assumptions underlying the mathematics of the model (and especially of the attrition processes), with the objective of fostering better understanding of all models.

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I. INTRODUCTION

VECTOR-2 is a deterministic simulation model of theater-level bilateral combat not involving biological, chemical, or nuclear weapons. It is intended for making net assessments, general purpose force tradeoff analyses, and studies of strategy and tactics. Our reaction to it is generally positive: it possesses impressive detail and flexibility and seems to be based on reasonable underlying assumptions.

This paper is a review and critique of the VECTOR-2 theater-level combat model and represents a continuation of previous work of the author reported in the following papers:

1) *On the VECTOR-1 Combat Model*, P-1181, Arlington, VA: Institute for Defense Analyses, 1976;
2) *On the Lulejian-I Combat Model*, P-1182, Arlington, VA: Institute for Defense Analyses, 1976;
3) *On the CONAF Evaluation Model*, P-1210, Arlington, VA: Institute for Defense Analyses, 1976;
4) *On the IDAGAM I Combat Model*, P-1265, Arlington, VA: Institute for Defense Analyses, 1977.

These papers are referenced as [8], [9], [10], and [12], respectively; the reader wishing to compare and contrast the various models is urged to consult these references. Also relevant, and complementary to the author's papers, is *Comparison and Evaluation of Four Theater-Level Models: CEM IV, IDAGAM I, Lulejian-I, VECTOR-1*, L.K. Walker, M.S. Higgins, and W.G. Svetlich, R-299, Arlington, VA: Weapons Systems Evaluation Group, 1976, referenced as [20].

Much of our analysis treats mathematical structures and assumptions underlying them, with further emphasis on attrition

processes in particular. While this concentration reflects biases, tastes, and capabilities of the author, it is also a serious attempt to facilitate understanding and comparison of theater-level combat models. Classical methods of evaluating and comparing models--such as using verifiably accurate data as inputs and then comparing outputs with empirical data, or testing by means of a controlled scientific experiment--are, for theater-level combat models, either impossible or extremely impractical. Moreover, a controlled experiment very likely would show only that no model is fully compatible with physical reality and hence would not provide a basis for making a choice among models in situations where one must be chosen for some particular analysis or study. An experiment might, however, show some models to be manifestly less realistic than others.

By treating underlying assumptions and mathematical structures we are able to describe some limitations of the different models, which is an essential prerequisite to using any of them; at least gross misuses may thereby be prevented. Moreover, once assumptions are understood, the model user has one rational basis for making comparisons and choices among models. Hence, even though some comparative comments below are stated as judgments, the objective has been to elucidate differences among models so that all may understand them better.

The remainder of this paper contains many references to the "three comparable models"--the CONAF Evaluation Model, the IDAGAM I Model and the Lulejian-I Model; we have not compared the VECTOR-1 and VECTOR-2 Models. The author's intention is to contribute to the common intellectual foundations of all the models and to greater understanding of the models both individually and relative to one another; criticisms are in this spirit throughout.

In preparation of this paper, our main source was the report [14], although [16] was frequently used to clarify points

not treated clearly in [14]. The reader is urged to consult [14] and [16] in conjunction with this paper.

2. ORGANIZATION AND GENERAL STRUCTURE

The VECTOR-2 model is structured as a feedback control system or, more precisely, as two interacting feedback control systems--one for each side. Operation of these systems, each of which contains several subsystems that are themselves feedback control systems, is described in more detail in Section 5. In broad terms, each decision maker compares perceived and desired states of the combat and takes actions designed to make the perceived state more like the desired state according to some criterion. The evaluative portion of the model, which computes attrition and movement, serves to define the actual state of the combat, of which the decision makers possess only imperfect knowledge. It should be noted that the property of being a "feedback control system" is not unique to the VECTOR-2 model, although as a means of explaining the model it is unique to the VECTOR-2 documentation. The three comparable models are also feedback control systems.

Figure 1 depicts the way in which model structure determines evolution of the combat over one time period. As usual, the two sides are called Blue and Red.

In representation of time, the VECTOR-2 model appears to differ significantly, although not necessarily to the extent implied by [14], from the CONAF Evaluation, IDAGAM I, and Lulejian-I models. The model incorporates six nested levels of fixed time steps which have the following proposed values in [14]:

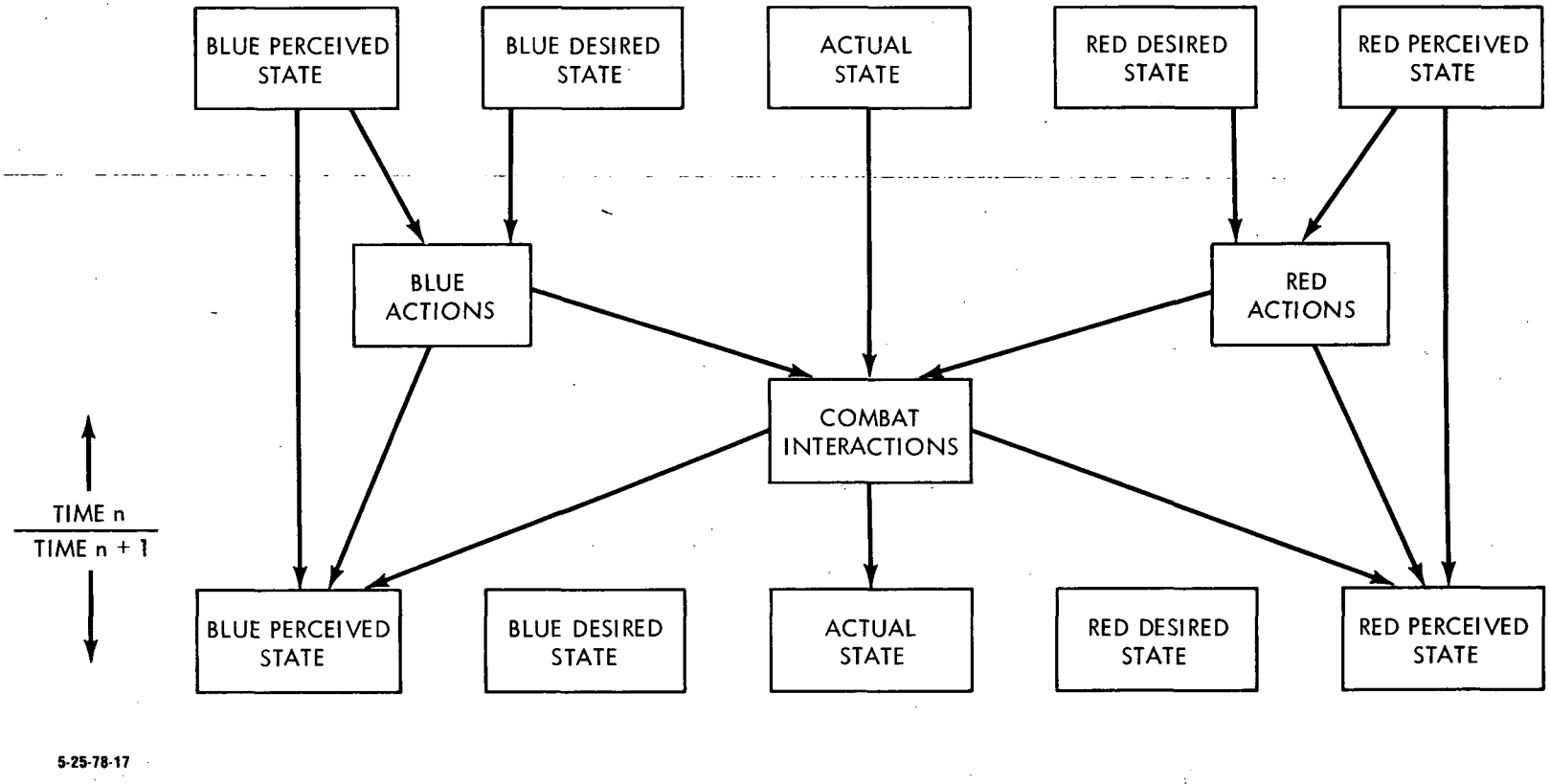

Figure 1. CAMPAIGN EVOLUTION IN VECTOR-2
(Schematic Representation)

LEVEL	PROPOSED INTERVAL
1	30 seconds
2	3 minutes
3	15 minutes
4	1 hour
5	6 hours
6	24 hours

The tabulation given above implies that Level 1 calculations, which represent front line combat using methodology described in Section 6, must be performed 2880 times per day of simulated combat. This may be a significant computational burden; cf. Section 10 for further remarks. In [16] rather larger time steps are proposed.

In addition to the set of nested time steps, the VECTOR-2 model contains a dual method of event scheduling. Events of long-term influence, such as force arrivals and aircraft mission assignments, take place at regularly spaced, artificial intervals, i.e., at the epochs of an appropriate time step level. On the other hand, events of short-term and spatially limited influence, such as maneuver unit close combat engagements, local movements of maneuver units, and arrivals of support fire are asserted to be scheduled "dynamically." This appears to be more an interpretation than an actual property of the model inasmuch as the computational structure of the model seems entirely time sequenced (cf. the flow chart on pp. 1-30 to 1-33 of [14]). Possibly what is meant is that events of short-term influence are assigned nominal times of occurrence, but that effects of these events are assessed at (for example) the next succeeding Level 1 time step.

Even so, use of extremely small time steps makes VECTOR-2 significantly different from the three comparable models in two respects. Obviously there is a greater level of detail, and hence possibly greater realism and flexibility, in VECTOR-2.

Less obviously, but perhaps more importantly, the fine time steps mitigate many of the problems with interaction sequencing that occur in other models because of arbitrary, albeit necessary, choices of orders of interactions. Over small time intervals, attrition is sufficiently small that spurious effects of such choices probably are negligible.

Somewhat surprisingly, given the level of detail of the model, there is no capability for day/night differentiation of effectiveness parameters and, indeed, essentially no capability to represent night time combat. The times when combat begins and ends each day are specified by input; it appears from [16] that one should not allow these to include the entire 24 hours, but probably only daylight hours.

Combat-related processes represented in VECTOR-2 may be put into the following categories:

1) Combat interaction processes (attrition)
2) Command and control processes
3) Intelligence and target acquisition processes
4) Communication processes
5) Logistics processes
6) Movement processes.

Each class of processes is discussed in detail in an appropriate Section below. The listing that follows gives the time step levels on which various process effects are calculated.

PROCESS	TIME STEP LEVEL
1) Theater-level processes	
a) Resource arrivals	6
b) Planning functions	6
2) Sector-level processes	
a) Resource arrivals	6
b) Intelligence processes	
• for theater-level and sector-level planning	6

PROCESS	TIME STEP LEVEL
• for front line maneuver units	3
• for other forces	4
c) Planning and mission and force allocations	
• for front line maneuver units	3
• for other forces	6
d) Reorganization, replacement, and resupply	6
e) Maneuver unit combat and movement	1
f) Target acquisition for support fire	
• for maneuver units as targets	2
• for other combat resources as targets	3
• for all other targets	5
g) Support fire allocation and effects	
• against maneuver unit targets	1
• against all other targets	3
h) Aircraft movement and interactions	1
i) Miscellaneous attrition and repair processes.	3

As a whole, the structure seems reasonably general and flexible. In particular, if it is possible to make all time steps the same size, in which case the nesting serves only to determine the order in which computations are performed, then VECTOR-2 could be reduced to a fixed time step model like the three comparable models. Note, however, that a change in the time steps requires concomitant changes in rates that are inputs to the model. Nonetheless, VECTOR-2 is structurally more general than the other models.

3. ENVIRONMENT: GEOGRAPHY AND WEATHER

In this Section we describe environmental aspects of the VECTOR-2 model, namely battlefield geography, terrain, and weather. Representation of these three aspects of combat is uneven: geography is unusually detailed and sophisticated for a theater-level model, with front-line maneuver units located in a real-space coordinate system. Terrain is represented parametrically in essentially standard fashion. Weather, one of the most important exogeneous, stochastic influences on combat, must be specified by input and affects evolution of the combat only through various indices of visibility and trafficability, as we will discuss in somewhat more detail. Weather intelligence is included as user-input five day forecasts for each side.

We shall describe fixed battlefield geography first in schematic form and then in terms of actual physical coordinates, after which "floating" geography, which is organizational and moves as forces do, will be treated. Figure 2 represents schematically the fixed battlefield geography.

Sectors are the largest geographic units in VECTOR-2 and are important in that combat cannot occur across sector boundaries; indeed, sectors are decoupled in the model to the extent that, except for theater-level calculations, only one sector at a time is treated in the core of the computer. At most seven sectors are permitted. From the standpoint of combat assessment, *combat arenas* are the principal geographic units in VECTOR-2; maneuver unit engagements do not extend across combat arena boundaries. A combat arena must be large enough to contain

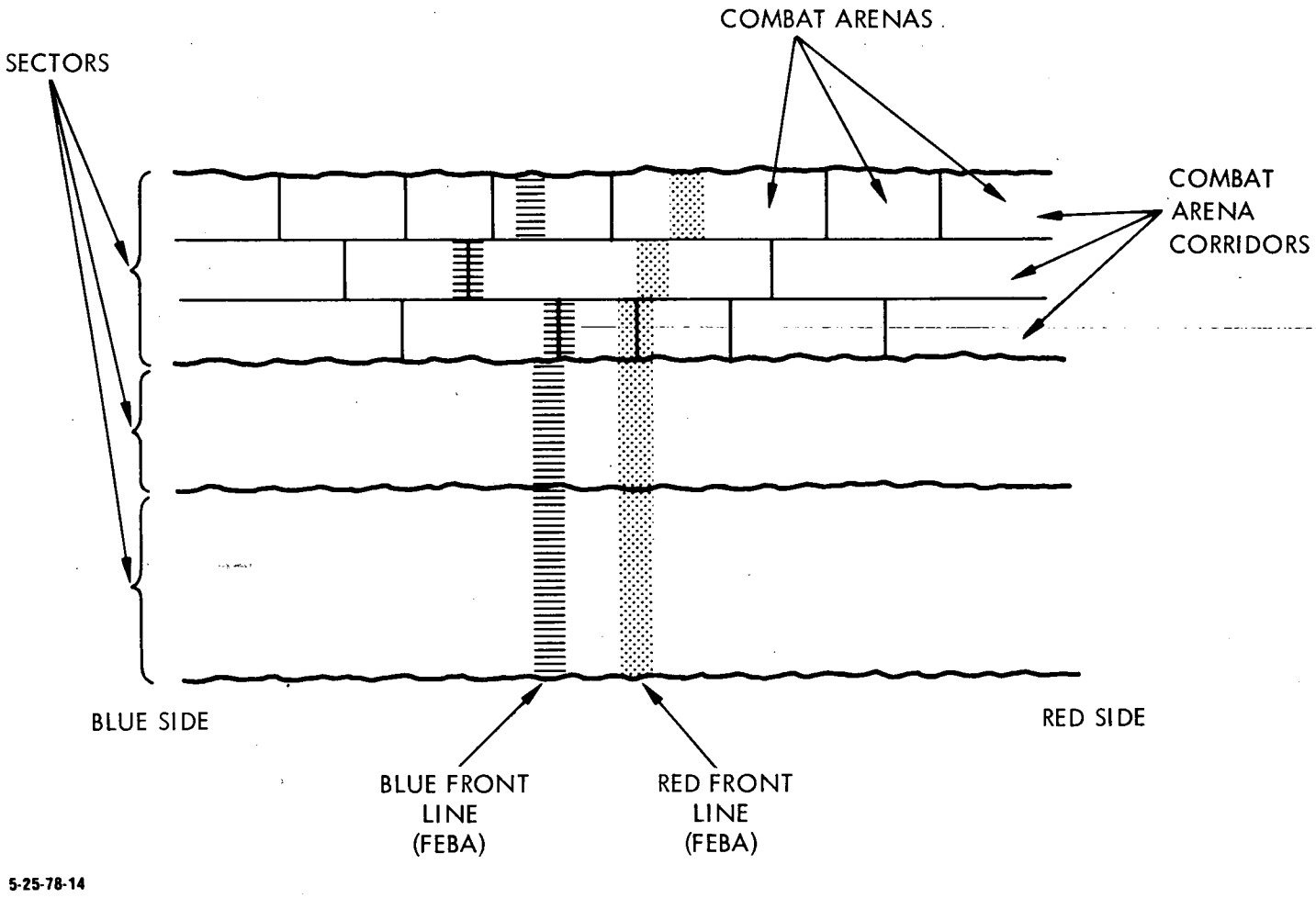

Figure 2. SCHEMATIC BATTLEFIELD GEOGRAPHY IN VECTOR-2

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a battalion-sized independent defense; its end boundaries represent terrain features (e.g., rivers) or changes in terrain type. It may contain arena-wide defensible positions, which are equally spaced but not closer than the range of direct fire weapons. The latter requirement ensures that opposing forces cannot enter a direct fire engagement when both are at defensible positions.

Combat arena corridors are strips of combat arenas that are adjacent front-to-back and run parallel to sector boundaries, as shown in Figure 1. Also as shown there, a combat arena has only one adjacent arena to its front and only one to its rear. A sector may contain no more than fifteen combat arena corridors.

Unlike those of comparable models, the FEBA in VECTOR-2 consists of two curves that represent the front lines of the Blue and Red sides, respectively. The FEBA need not coincide with the schematic geographical boundaries in the model; it may even run through a combat arena as shown in Figure 2 (in other places the FEBA does coincide with arena boundaries). In each combat arena corridor, FEBA position is schematically constant and the opposing front lines are separated by a positive distance. FEBA position over a sector need not be constant. As discussed in Section 8, FEBA movement is computed within the model from physical movement data and not, as in CEM, IDAGAM I and Lulejian-I, from artificially devised tables or systems of equations.

In the three comparable models, schematic geography and actual geography are virtually indistinguishable; in VECTOR-2 schematic geography is truly schematic. The model contains an explicit coordinate geography in terms of which aircraft and maneuver units are located. An example is given in Figure 3. Where, for simplicity, only the topmost sector of Figure 2 is repeated. Clearly, specification of combat arena boundaries entails significant effort.

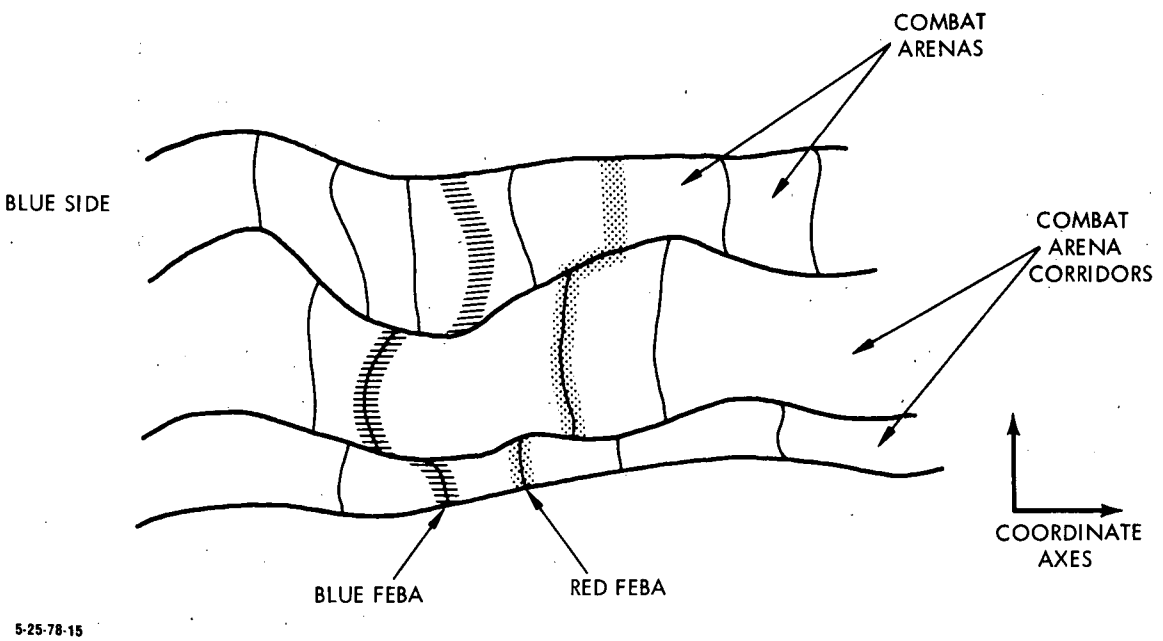

Figure 3. COORDINATE BATTLEFIELD GEOGRAPHY IN VECTOR-2

The geography discussed above is representative, either schematically or exactly, of actual battlefield geography; in addition VECTOR-2 incorporates an organizational geography that moves as the FEBA does and serves to locate less important resources and in resource allocation. Figure 4 depicts organizational geography; for simplicity the organizational geography is superimposed on the schematic geography of Figure 2 and shown only for one sector and only for the Blue side.

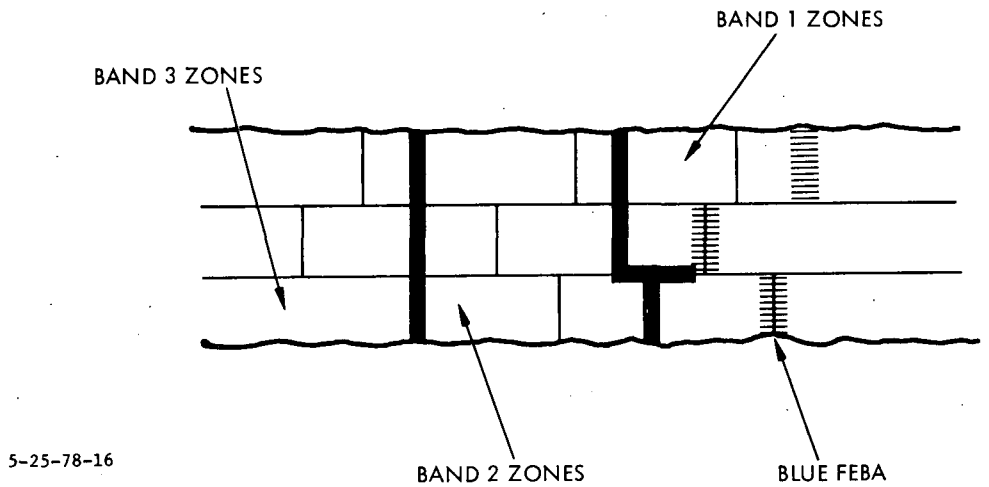

Figure 4. ORGANIZATIONAL BATTLEFIELD GEOGRAPHY IN VECTOR-2

Floating, organizational geography is represented in VECTOR-2 by means of *zones* that move with the FEBA and serve to define various rear regions. Lateral boundaries of a zone are specified combat arena corridor boundaries, while the back boundary is a prescribed distance from the front boundary. Four bands of zones are permitted. There need not be symmetry or alignment of zone boundaries on the two sides or in different corridors on the same side.

Within this system of geography, front line and reserve maneuver units are represented in actual coordinate locations, as are airborne flights of aircraft. The following combat resources are located only in terms of zones by assuming them to be at characteristic and user-specified distances from the FEBA: field artillery batteries, air defense sites, observation resources, supply depots, and air bases. Particularly for aircraft in flight and air defense sites, this creates asymmetries that do not represent physical reality.

Terrain type in VECTOR-2 is defined by specifying one of six levels of intervisibility (which affects the attrition computations described in Section 6) and one of six levels of trafficability (which influences movement computations); hence there are a total of 36 different terrain types, although presumably not all combinations of trafficability and intervisibility are equally plausible. Each combat arena contains terrain of only one type. Terrain features represented in the model include urban areas, rivers, and one user-defined feature (e.g., mountains); these occur at ends of combat arenas, are at least arena-wide, and require special movements and possible attacks in order to be passed.

Weather is described by specifying one of four levels for each of the following five parameters:

1) visibility for ground-to-ground operations

2) visibility for air-to-ground and ground-to-air operations
3) visibility for air-to-air operations
4) trafficability for ground operations
5) trafficability for air operations.

These parameters are specified by input on a per-sector, per-hour basis and cannot be neglected (for reasons of simplicity or efficiency, for example). Also, air-to-ground visibility and ground-to-air visibility are unjustifiably taken to be the same.

From terrain and weather data the model computes for each combat arena an environmental trafficability index and an environmental visibility index, which are updated each hour. Both indices have parametric effects on attrition and movement computations.

4. RESOURCES AND LOGISTICS

The following is a list of resources modeled in VECTOR-2 and missions those resources may undertake:

1) Ground Forces

a) Maneuver units

Resources: 11 weapon types, personnel of 1 type, minefields

Missions: Pursue withdrawing enemy, advance unopposed, counterattack, attack/defend in urban area, attack/defend at user-specified terrain feature, attack/defend at river, attack/defend at normal defensive position, delay, withdraw under attack, withdraw not under attack, regroup, be inactive.

b) Artillery and mortar forces

Resources: 5 weapon types, personnel

Missions: pre-engagement fire, final protective fire, disengagement fire, fire at acquired targets

c) Air defense sites

Resources: 6 weapon types, personnel

Mission: fire against aircraft and helicopters

2) Tactical Air Forces

a) Aircraft

Resources: 7 aircraft types, personnel, 3 types of shelters (2 of which hold only a specified type of aircraft)

Missions: CAS, escort, intercept, attack of acquired ground targets (reserve units command posts, supply depots, field artillery batteries, air defense sites, air bases, observation resources)

b) Attack helicopters

Resources: one type only, personnel

Missions: fire support, use as front line maneuver unit weapons

3) <u>Observation Resources</u>

Resources: 14 types of sensors

Mission: Acquire targets for fire support weapons

4) <u>Command Posts</u>

5) <u>Supplies</u>

Resources: Ammunition for each type of maneuver unit weapon, land mines, 11 types of ordnance for aircraft and helicopters, aviation POL, ground force POL, one other category of supplies

Virtually all the missions mentioned above are discussed in more detail in Sections that follow and will not be elaborated upon here. It appears that weapon types within a category are distinguished for accounting purposes and only on the basis of differing values of numerical parameters. The numbers of types of weapons seem generally adequate but cannot easily be changed, although the Program Change Monitor discussed in Section 10 makes such an undertaking less hazardous than for the CEM and Lulejian-I models. In IDAGAM I, the process requires only a change in certain inputs.

Command posts seem to serve two roles in VECTOR-2. First, damage to a command post decreases responsiveness of associated forces; second, command posts constitute an accounting device for representing communication delays. See Section 5 for details.

The resources listed above are organized into resource groups with differing combat characteristics. Types and numbers of types of resource groups are the following:

1) Aggregate groups
   a) Artillery batteries: 5 types
   b) Attack helicopter bases: 1 type
   c) ADA sites: 4 types
   d) Sensor groups: 4 types
   e) Supply groups: 3 types
   f) Other groups: 3 types
2) Airfield groups: 3 types
3) Maneuver unit groups: 5 types, including command posts
4) Air groups: 7 types

Organization, force structures, and hierarchies are prescribed by the tactical decision rules of the model; the latter are discussed in Section 5 below. Depending upon the capabilities and resources of the user, force structures and hierarchies are almost unrestricted; very intricate and detailed command structures can be constructed.

Logistics processes in the VECTOR-2 model perform storage and distribution of resources, specifically:

1) Personnel
2) Weapon systems (of types mentioned above)
3) Supplies of 34 types as follows:
   a) 8 types of ordnance for maneuver unit weapon systems
   b) 5 types of ammunition for artillery and mortars
   c) 6 types of ammunition for air defense weapons
   d) 11 types of ordnance for aircraft and attack helicopters
   e) mines
   f) POL for ground vehicles
   g) POL for aircraft and helicopters
   h) 1 general class of supplies.

Supplies enter either the theater or a user-designated sector and may be stored at the sector level or by user-specified

resource groups. Distribution of supplies is computed by tactical decision rules.

With tactical decision rules, the user can construct representations of supply shortages, including the following:

1) zero-one phenomena such as immobilization of resource groups without POL or inability to fire of weapons without ordnance;
2) linear degradations based on supply shortfalls;
3) no effect on resource function, but with supply inventories still computed (and allowed to become negative).

Supplies are depleted by consumption and attrition resulting from enemy fire. In general, supply consumption is a linear function of some state variable that represents the level of activity of a resource. Hence, for example, aviation POL consumption C_A is computed using an equation of the form

$$C_A = c_A(\sum_i s_i) + c_A' p \ , \tag{4.1}$$

where

s_i = length of combat sortie i,
c_A = POL consumption rate, as a function of distance, for combat sorties,
p = number of personnel in sector,
c_A' = POL consumption rate, per time period per person, for noncombat uses.

Similarly, each aircraft sortie consumes a user-specified load of ordnance, which may depend on the type of aircraft and the mission. POL consumption by attack helicopters is proportional to time spent in the air.

For weapon systems in maneuver units, ordnance consumption is proportional to the firing rate; the latter is determined by an equation of the form

$$\rho_i = \rho_i^0 \sum_j f(i,j) \ , \tag{4.2}$$

where

ρ_i = firing rate for weapon of type i,

ρ_i^0 = firing rate when actually firing,

$f(i,j)$ = fraction of time spent firing at targets of type j.

In general $\sum_j f(i,j) < 1$ so that ρ_i accounts for time when the weapon is not firing, whereas ρ_i^0 does not.

The report [14] does not elaborate upon attrition to supplies. Presumably, and very reasonably, the scheme is rather simplistic.

Only limited functions of construction, repair, and maintenance are represented in VECTOR-2, which seems quite justifiable in a model intended primarily for analysis of combat effects only, but not in a model designed to study the participants' capabilities to support their forces. Fixed fractions of damage to ADA sites and command posts are repairable each Level 4 and Level 2 time period, respectively. New air bases can be constructed using tactical decision rules and overrun air bases can be reconstructed, both at user-input rates. Construction of aircraft shelters is effected through tactical decision rules. No repair of maneuver unit weapons appears possible.

5. COMMAND, CONTROL, AND COMMUNICATION PROCESSES

The command, control, and communication processes in the VECTOR-2 model seem sensible and appropriate for a theater-level model, unlike those of the Lulejian-I and IDAGAM I models, which are nonadaptive, inflexible, and over-simplistic, and those of the CONAF Evaluation model, which are so detailed that they overwhelm the rest of the model. Some effects included in VECTOR-2 without great error might be neglected, but the representations seem to depict phenomena of importance in determining the short-term evolution and ultimate outcome of a battle. Moreover, the level of detail is generally consistent with the rest of the model.

In the conceptual sense, the command, control, and communication processes in VECTOR-2 constitute a feedback control system. Each commander is presented with:

1) A *desired state* of the combat, represented by state variables defining mission, objectives, available resources, and so on, values of some of which may have been set by decisions at higher levels;
2) A *perceived state* comprised of intelligence concerning enemy forces, environmental and scenario data, and (possibly out-of-date) information about friendly forces.

Based on the perceived state, the commander undertakes actions intended to change the perceived state into the desired state.

It is claimed in [14] that the model represents decision making "from squad to theater level," but it seems that only for higher levels of command is the representation reasonably complete and accurate. Differences among levels are the size of the system that can be affected, the resources available, and the frequency with which decisions can be altered.

The three decision-related functions may be defined as follows:

1) Command: mission assignment and task force configuration
2) Control: dynamic allocation of resources
3) Communication: information flow and delay.

Command and control are represented primarily by the tactical decision rules described in more detail below in this Section. Each decision is defined by:

1) A point in time at which the decision must be taken
2) Outputs that must be prescribed by the decision
3) Information used as input
4) Method of computing outputs.

Communication is represented by delays in implementation of command and control decisions.

The command hierarchy is user-specified, subject only to the constraint that no unit have more than one direct superordinate; the scheme is advantageous in terms of flexibility but imposes substantial preparations that must be undertaken before running the model. Only maneuver units and their superordinates are included, which is not a serious restriction. Indeed, the lowest level in the hierarchy is the battalion level, at which, appropriately, effects of direct fire engagements are computed. One then questions, however, whether the model really represents decision making "at the squad level." The command hierarchy may change over time, a feature not included in the three comparable models.

To aid in mediation of conflicting resource requirements of proximate maneuver units, an objective overlay exists that characterizes interdependence of objectives in pairs of adjacent combat arenas, contains a representation of terrain, and is used in task force construction, mission assignment,

establishment of forward combat arenas in each sector, and allocation of arena ribbons to brigades or divisions.

With most models, the greatest potential strength is, if not carefully exploited, possibly the greatest weakness and so it is with the tactical decision rules in VECTOR-2. In terms of potential for--

1) providing a systematic and unified treatment of decision making,
2) flexibility,
3) ease of variation, and
4) realism and detail--

the tactical decision rule structure of VECTOR-2 easily surpasses even the decision making structure of the CONAF Evaluation model. But the word "potential" is crucial; to be effective, tactical decision rules must be intelligently and carefully constructed so that the user is certain that effects desired to be represented are in fact represented. If misused or abused, the tactical decision rules render the model and its outputs untrustworthy.

In terms of decision making structure, the model has fixed within it only

1) points in time and command levels at which decisions are required, and
2) for each decision, a minimal set of required outputs, namely state variables whose values must be set before the model can proceed (other state variables can be set or modified if desired).

The user can and must specify, in the form of a FORTRAN subroutine, the method for evaluating required (and any additional) outputs. All state variables may be used as inputs, but care must be exercised to prevent inadvertent changes in values of the state variables.

Inputs to a particular tactical decision rule may be grouped into three classes:

1) Those representing the desired state of the combat (e.g., a mission assignment by a higher echelon)
2) Those representing the perceived state of the combat
3) Data used in comparison of perceived and desired states and computation of outputs.

A simple example is given in [14, pp. 79-82].

The following is a complete list of tactical decision rules in the VECTOR-2 model, along with their subroutine names:

1) *Theater level* tactical decision rules for:
   a) Distribution of theater arrivals to sectors, and distribution of groups and resources among sectors (TRTDIS)
   b) Assignment of missions to sectors (TRTMIS)
   c) Construction of new air bases and shelters (TRBILD)
   d) Assignment of aircraft to air bases (TRADIS)
   e) Assignment of aircraft to air groups and air groups to sectors (TRAGAL)
   f) Setting times for preplanned air missions (TRTIMP)
   g) Setting targets for preplanned air missions (TRAGAZ)
   h) Calculation of rate-based attrition to maneuver units in transit between sectors (TRUKIL).

2) *Sector level* tactical decision rules for:
   a) Assignment of front line responsibilities to maneuver units (TRFRNT)
   b) Reorganization of maneuver units (TRREOR)
   c) Construction of command hierarchy (TRDAD)
   d) Distribution of replacements to resource groups (TRDIST, TRDOLE)
   e) Assignment of nonorganic artillery to maneuver units (TRRART)
   f) Shifts of independent resource groups and artillery among zones (TRSHFT, TRREDO)
   g) Allocation of some air groups assigned to the sector to CAS mission (TRAGAC, TRCASD)
   h) Allocation of air groups to targets (TRACAL)

i) Selection of secondary targets for air missions (TR2TGT)

j) Setting conditions for aborting air missions (TRAAAI)

k) Allocation of attack helicopters from front line divisions to front line maneuver units (TRAHAL)

l) Allocation of air groups with interdiction mission to targets (TRINT)

3) *Battalion task force level* tactical decision rules for:

a) Assignment of missions to subunits of a maneuver unit (TRMISS)

b) Readying a maneuver unit for combat by restructuring its subunits (TRREOR)

c) Allocation of organic attack helicopters from front line maneuver unit to subunits (TRAHGD)

d) Target allocation for air-to-air combat (TRAATA)

e) Specification of choice of action after Phase 1 of air-to-air combat (TRPH1)

f) Specification of choice of action after Phase 2 of air-to-air combat (TRPH2)

g) Determination of possible breakoff by attacking aircraft after each pass on a ground target (TRATK)

h) Determination of maneuver unit close combat decisions, including:

- Choice of activity (posture)
- Calls for commitment or reconstitution of reserves
- Choice of action when confronted by a minefield
- Calls for support fire (TRSITN)

i) Selection of allocation method for normal support fire (TRSPLF)

j) Possible alteration of support fire allocation priorities (TRPRIT)

k) Allocation of support fire to acquired targets (TRFSAL)

l) Calculation of rate-based attrition to target acquisition resources (TRSKIL).

A total of 34 tactical decision rules is required.

For the user who wishes to construct his or her own set of tactical decision rules, significant effort is entailed. However, a demonstration package of tactical decision rules is available from the model developers, which provides a chance to use the model immediately and a convenient starting point from which tactical decision rules can be constructed that represent specific phenomena of interest. Tactical decision rules in this package are described in [16]. Although [14] is not clear about the point, we believe that the two sides need not use the same tactical decision rules.

As noted previously, communications effects in VECTOR-2 are manifested only as delays in transmitting messages between the battalion, regiment, division, and corps levels. Messages may be either priority or nonpriority, but a priority message does not preëmpt a nonpriority message. The model employs the following equations to compute expected delays for messages originating at a given node with a given destination. Let

W_p = expected waiting and transmission time for a priority message, and

W_n = expected waiting and transmission time for a nonpriority message

be the outputs to be calculated and let

λ = rate of message generation at a given node (which depends on whether combat is in progress nearby),

μ = single channel message transmission (completion) rate, and

s = number of channels in communication link.

Then, provided $\lambda < s\mu$, the relevant expressions are

$$W_p = \frac{q}{s\mu(1 - \frac{\lambda}{s\mu})} + \frac{1}{\mu}, \tag{5.1}$$

and

$$W_n = \frac{q}{s\mu(1 - \frac{\lambda}{s\mu})^2} + \frac{1}{\mu}, \tag{5.2}$$

where q is the limiting probability (as $t \to \infty$ and the system reaches equilibrium) that all channels are in use but no message is currently awaiting transmission.

The reviewer cannot derive these equations from the assumptions stated in [14]; at a minimum, the fraction of messages that are priority messages must be specified.

6. ATTRITION IN GROUND COMBAT

In this Section we describe the most complex and sophisticated set of calculations in the VECTOR-2 model: those used to compute ground combat losses. To a significantly greater extent than those of the comparable CONAF Evaluation, IDAGAM I, and Lulejian-I models, the ground combat attrition methodology in the VECTOR-2 model is based on physically defined and measurable input parameters and on environmental conditions. While the other models might represent implicitly in the preparation of data bases such phenomena as existence of lines of sight, range, terrain, weather, and different methods of target acquisition, only VECTOR-2 can do so explicitly and dynamically. The reviewer believes that VECTOR-2 is without question superior to the other three models in representation of this extremely important component of theater-level combat. Further general comments on ground combat attrition computations in VECTOR-2 appear in Section 11.

We begin by describing the VECTOR-2 methodology for assessing effects of engagements between opposing maneuver units, which are typically battalion task forces. The reader uninterested in mathematical details may wish to omit pages 38-47.

Maneuver unit engagement effects are computed individually for each combat arena and calculated once each Level 1 time step (cf. Section 2). Explicit representation of the following phenomena is included:

1) Maneuver unit strengths and coordinate locations
2) Weapon system performance data
3) Environmental conditions: weather and terrain

4) Weapons deployment and movement
5) Mounting and dismounting of infantry from APCs
6) Open- and cease-fire ranges
7) Two methods, parallel and serial, of target acquisition and selection by shooting weapons
8) Existence of a line of sight as a necessary condition for target acquisition.

Every shooting weapon is assigned a list of priorities of targets. These priorities do not represent physically measurable quantities and are judgmentally derived in most cases; to the extent that the priorities influence results obtained using the model, those results may be suspect.

A weapon with *parallel target acquisition* continues to search for new targets while engaging a target and, should a target of higher priority be acquired, will immediately (and instantaneously) switch its fire to that target. A weapon with *serial target acquisition* will, once it engages a target, continue to engage it until the target is destroyed (possibly by another shooting weapon) or the line of sight to the target is lost; during an engagement the shooting weapon does not seek to acquire further targets. The method of target selection, however, is quite complicated and is specified by a sequence of *search cutoff times*. Assume that there are N types of weapons on the opposing side; for a particular type of shooting weapon there will then be N-1 search cutoff times

$$t_1 < t_2 < \ldots < t_{N-1} .$$

If a target of priority 1 is acquired before t_1 (measured from the beginning of an acquisition effort) it will be engaged at once. During this time, targets of lower priorities may become and remain acquired but will not be engaged. If no priority 1 target is engaged before t_1, but a priority 2 target has been acquired and remains visible at t_1, it then will be engaged immediately. If neither of these situations has occurred, the

shooting weapon continues to search for targets. If in the time interval (t_1, t_2) a target of priority 1 or priority 2 is acquired, it will be engaged at once. Should this not happen, any target of priority 3 acquired in $[0, t_2]$ and still visible at t_2 then will be engaged at t_2; otherwise the search continues and any target of priority 1 or 2 or 3 acquired in (t_2, t_3) will be immediately engaged, and so on. The following table summarizes.

Table 1. RULES FOR SERIAL TARGET SELECTION IN VECTOR-2

Time	Priorities of Targets to be Engaged Immediately Upon Acquisition	Priorities of Targets to be Engaged if Previously Acquired and Still Visible
$(0, t_1)$	1	
t_1		2
(t_1, t_2)	1,2	
t_2		3
(t_2, t_3)	1,2,3	
⋮ (t_{N-2}, t_{N-1})	⋮ 1,...,N-1	⋮
t_{N-1}		N
(t_{N-1}, ∞)	1,...,N	

As noted in [14, p. 1-42], important assumptions that underlie the ground combat attrition computations in VECTOR-2 are:

1) Destroyed and undestroyed targets are distinguishable with certainty; a previously destroyed target is never engaged. Destruction of a target is immediately discernible to *all* shooting weapons.
2) All weapons of the same nominal type in the same location are equally vulnerable and equally effective.

3) For a given shooting weapon and target, lengths of visible periods (when a line of sight exists) are independent and identically exponentially distributed; lengths of invisible periods are independent and identically exponentially distributed (possibly with a different expectation) and independent of lengths of visible periods. Visible and invisible periods alternate.

4) Given continuous existence of a line of sight, the time required to acquire a target is exponentially distributed.

5) Given continuous acquisition and target survival of attacks by other weapons, the time necessary to destroy a target is exponentially distributed.

6) For a given shooting weapon, *all* visible and invisible periods, acquisition times, and times-to-kill are mutually independent.

7) The entire line of sight, acquisition, and kill processes of different shooting weapons are mutually independent, except that a currently engaged target may be destroyed by another shooting weapon.

8) There is no extraneous damage (to other, nearby targets that are not under engagement, e.g.).

Parameters above depend upon types of both shooting and target weapons and possibly also on environment and range.

Given the level of detail of the ground combat portion of the VECTOR-2 model, Assumption 1) seems unjustifiably restrictive. There should be simple, reasonable alternatives.

Assumptions 6) and 7) are crucial in physical and mathematical terms. Physically, they imply that a shooting weapon is never aided in acquisition of one target by acquisition or destruction of another target and that shooting weapons on a given side act without coordination or synergistic effects, which may be viewed as a limitation of the model, albeit a nearly universal limitation. Furthermore, no shooting weapon may simultaneously engage more than one target, and acquisition of a target is not influenced by the fact that the target itself may be firing upon the shooting weapon. However, despite these somewhat unrealistic physical implications, the assumptions are

probably as complex as the current state of knowledge concerning stochastic attrition processes allows; the same assumptions, cf. [9,10,12], are present in the CEM and the IDAGAM I and Lulejian-I models.

Mathematically, as demonstrated and discussed in [7,11], Assumptions 6) and 7) determine the qualitative structure of the VECTOR-2 attrition process and, consequently, of the attrition equation [(6.1) below] chosen to approximate relevant expectations. Because of serial target selection, the stochastic attrition process engendered by Assumptions 1) - 8) above is not Markov, but is a semi-Markov process. The reader is referred to [4,5] for background and details concerning semi-Markov processes. Before we discuss the attrition process in more detail, we introduce necessary notation.

We describe attrition to weapons in a maneuver unit on the Red side during one (Level 1) time period; calculations for attrition to the Blue side are completely analogous. Let

$R_j(t)$ = number of type j weapons on Red side at time t,

$B_i(t)$ = number of type i weapons on Blue side at time t,

Δt = Level 1 time increment.

These numbers of weapons do not include weapons devoted to flank defense, weapons without ammunition, or (if a side is delaying) weapons at the next defensible position to the rear. Additional notation will be defined as necessary.

Assumptions 6) and 7) imply that expected losses of Red weapons of type j during the time interval $(t,t+\Delta t]$ are given approximately by

$$\Delta R_j(t) = R_j(t) - R_j(t+\Delta t) = [\sum_i a(i,j,B(t),R(t))B_i(t)]\ \Delta t\ . \tag{6.1}$$

The function a is the *attrition rate function* and depends on the current structures of the Blue and Red forces; its derivation is discussed below. Were the underlying attrition process Markov, derivation of (6.1) would be straightforward, based upon well-known properties of infinitesimal generators. In the semi-Markov case, more care and additional assumptions are required.

In terms of known and studied attrition models, (6.1) is not easy to classify. At first glance, the process appears, in the terminology of [11], to be of independent engagement initiation form, but only if one ignores (improperly) dependence of the attrition rate function on B(t) and R(t). One class of deterministic analogues of (6.1) is the family of "variable coefficient Lanchester models" extensively studied by S. Bonder and J. Taylor (cf., for example, [3,4,19]). Note that (6.1) can also be viewed, cf. [7,11], inaccurately but appealingly, in terms of interpretation and understanding, as arising from a *Markov* attrition process with infinitesimal generator A given by

$$(6.2) \qquad A[(x,y),(x;y_1\ldots,y_j-1,\ldots,y_N)] = \sum_i a(i,j,x,y)x_i$$

where $x \in R^M$ represents Blue forces, $y \in R^N$ represents Red forces (M and N are the numbers of Blue and Red weapon types, respectively).

Indeed, in some ways it seems to the reviewer that this latter interpretation of (6.1) is the most reasonable, even though it is not strictly compatible with Assumptions 1) - 8) above. In this interpretation, (6.1) is the standard approximation based on infinitesimal generators, and calculation of the attrition rate function as described below amounts to a careful, sophisticated, and reasonable derivation of the infinitesimal generator of the Markov attrition process.

A further complication is that the attrition rate function incorporates parameters (cf. (6.3), for example) that depend

on environmental factors and hence also on time; the resulting stochastic attrition process is, consequently, not temporally homogeneous. Nonhomogeneous semi-Markov processes are not well understood, while nonhomogeneous Markov processes, although not amenable to detailed computational treatment, are at least fairly well understood. Moreover, even in the nonhomogeneous Markov case, the interpretation and validity of (6.1) remain as previously discussed.

We now discuss derivation of the attrition rate function. The type i of shooting weapon will hereafter be fixed and dropped from the notation; all parameters below depend on it. Assume first that the shooting weapon employs parallel target acquisition; this is easier to handle than serial acquisition. Without loss of generality we suppose that opposition weapon types and priorities (for the given type of shooting weapon) coincide, which will simplify our exposition.

The data (model inputs) used to compute $a(j,B(t),R(t))$ in this case are for each target type k:

μ_k = exponential parameter for lengths of visible periods,

η_k = exponential parameter for lengths of invisible periods,

λ_k = exponential parameter for time to acquire given continuous visibility,

α_k = exponential parameter for time to kill given continuous acquisition and engagement.

The parameter of an exponential distribution is the inverse of its expectation, i.e., the rate of the associated Poisson process. These parameters may vary over time and may depend on the distance from the shooting weapon to the target.

The VECTOR-2 model incorporates the following expression, based on standard limiting results in renewal theory, for the attrition rate function:

$$(6.3)\quad a(j,B(t),R(t)) = \alpha_j\left[1-\left(1-\frac{\eta_j}{\mu_j+\eta_j}\,\frac{\lambda_j}{\mu_j+\lambda_j}\right)^{R_j(t)}\right]$$

$$\times\prod_{k=1}^{j-1}\left(1-\frac{\eta_k}{\mu_k+\eta_k}\,\frac{\lambda_k}{\mu_k+\lambda_k}\right)^{R_k(t)}.$$

The interpretation of (6.3) is both natural and reasonable:

1) For each k, $\eta_k/(\mu_k+\eta_k)$ is the probability that a given target of type k is visible;

2) For each k, $\lambda_k/(\mu_k+\lambda_k)$ is the probability of acquisition given visibility.

Therefore the indexed product on the right-hand side of (6.3) is the probability that no target of priority less than j is visible and acquired; the second factor is the probability that at least one type j target is visible and acquired; the product is [by Assumption 6)] the probability that the shooting weapon is currently engaging a target of type j. Multiplied by the rate of kill given engagement, this quantity becomes the appropriate attrition rate.

Observe that (6.3) ignores the possibility that a currently engaged target may be destroyed by some other shooting weapon. To the reviewer this seems a reasonable simplification; it is not, however, incorporated into attrition rate functions for shooting weapons with serial target acquisition, the case we describe next.

In the following discussion the type i of the serial acquisition shooting weapon is fixed throughout and suppressed from the notation; unless explicitly stated to the contrary, each parameter mentioned is a function of the type of shooting weapon. We assume, without loss of generality, that target type and target priority for opposition weapons coincide. Data

for the computation, provided by the user as inputs to the model, are:

$t_1 < \ldots < t_{N-1}$ = search cutoff times (cf. page 28 for an explanation),

μ_k = exponential parameter for visible periods of type k target weapons,

η_k = exponential parameter for invisible periods of type k target weapons,

λ_k = exponential parameter for time to acquire a type k target weapon, given continuous visibility,

α_k = exponential parameter for time to destroy a type k target weapon, given continuous engagement.

Factors which the attrition rate function $a(\cdot,\cdot,\cdot)$ explicitly represents are search cutoff times, loss of line of sight, and possible destruction of the target by another shooting weapon. As previously noted, the latter is ignored in parallel target acquisition; it would seem reasonable and more consistent to ignore it in this case as well.

Computation of the function a involves modeling the time-evolving state of a particular shooting weapon as a semi-Markov process $(Y_t)_{t \geq 0}$. For details concerning semi-Markov processes and Markov renewal processes, from which they arise, the reader is referred to [4,5]. Except for an important limit theorem, none of the results developed in [4,5] is needed in the VECTOR-2 model. The state space E of this semi-Markov process is given by

$$E = \{k_1, \ldots, k_N, \ell_1, \ldots, \ell_N\} ,$$

where

k_j = state of ultimately acquiring or actively engaging a type j target that will be destroyed by this engagement,

ℓ_j = state of acquiring or engaging a type j target that will not be destroyed by *this* engagement (because the line of sight is lost or the target is destroyed by another shooting weapon).

Let P be the transition matrix of the Markov chain embedded in (Y_t), i.e., the sequence of states entered by (Y_t) without reference to the time spent in each, and let $\overline{G}(x)$ be the expected length of a visit of the process (Y_t) to state x. By standard results for Markov chains, which are applicable at least if all types of opposition weapons are present, there exists a unique probability distribution ν on E such that

$$\nu = \nu P \ . \tag{6.4}$$

Then, by an important limit theorem for semi-Markov processes (cf. Theorems (10.4.3) and (10.5.22) of [5]),

$$\lim_{t\to\infty} P\{Y_t = x\} = \nu(x)\overline{G}(x)\Big[\sum_{y\in E} \nu(y)\overline{G}(y)\Big]^{-1} \tag{6.5}$$

for each $x \in E$. The attrition rate function is then taken to be

$$a(j,B(t),R(t)) = \nu(k_j)\Big[\sum_{y\in E} \nu(y)\overline{G}(y)\Big]^{-1} \ . \tag{6.6}$$

Roughly speaking, the right-hand side of (6.5) is for $x = k_j$ the probability of being in the state k_j, and $\overline{G}(k_j)$ is the expected time spent in k_j, at the end of which the target is actually destroyed, so the right-hand side of (6.6) is the *rate* at which the target is being destroyed. Note that the interpretation is based on infinitesimal behavior of the process, while (6.5) and (6.6) are based on limiting behavior.

There are certainly intellectual, and possibly practical, difficulties describing manifestly transient behavior of a process, which one wants to incorporate in differential models of combat, by means of limit theorems. A further difficulty

is that the transition parameters of the semi-Markov process (Y_t) depend on current force structures B(t) and R(t) that are themselves (random) functions of time; invocation of limit theorems that assume these to be constant is then inappropriate.

Because alternatives have not been developed and explored, the magnitude and implication of practical difficulties arising from use of (6.6) cannot be assessed. An alternative that more realistically represents transient aspects of the attrition rate function would be to take a(j,B(t),R(t)) to be the inverse of an appropriately defined expected first passage time to a (newly defined) state of "engaging a type j target that will be destroyed by this engagement."; cf. [17,18] for a more complete discussion of the relevant, but unapplied, mathematics.

The problem of computing the attrition rate function a is now reduced to computing the transition matrix P and the expected sojourn lengths $\overline{G}(x)$, $x \in E$. We shall deal with them in that order. First, for all $x \in E$ and each j

$$P(x,k_j) = q_j h_j$$
$$P(x,\ell_j) = q_j(1-h_j) \quad , \tag{6.7}$$

where

q_j = probability that the next target selected for engagement is of type j,

h_j = probability that a currently engaged target of type j is destroyed before loss of line of sight or destruction by another shooting weapon.

The numbers q_j, h_j must now be computed.

In the VECTOR-2 model, the probability q_j is computed by the equation

$$(6.8) \qquad q_j = d_j(t_{j-1}) \prod_{r=1}^{j-1} [1-H_r(t_{j-1}-t_{r-1})][1-d_r(t_{r-1})]$$

$$+ (1-d_j(t_{j-1})) \sum_{r=j-1}^{N}$$

$$\int_{t_r}^{t_{r+1}} \left(\prod_{\substack{s=0 \\ s\neq j-1}}^{r} [1-d_{s+1}(t_s)][1-H_{s+1}(x-t_s)] \right) H_j(dx-t_j) \ ,$$

where

$d_k(u)$ = probability that a target of type k has been and remains acquired u time units after initiation of search,

$H_k(\cdot)$ = distribution function of time to acquire a target of type k.

One must interpret (6.8) term by term:

1) The first factor in the first summand is the probability that a target of type j is acquired before t_{j-1} and hence eligible for selection at t_{j-1}; the product is the probability that no target with greater priority has previously been selected.

2) In the second summand, $1-d_j(t_{j-1})$ is the probability that a given target of type j is not acquired at t_{j-1} and hence not immediately available for selection then. The other factor represents the probability that no other target (of any priority) is selected first; r+1 is the type of the lowest priority target for which search is actually initiated.

The remaining calculations are not hard to describe; to compute $H_k(\cdot)$ the model uses the equation

$$(6.9) \qquad H_k(u) = 1 - \exp\left(- \frac{\lambda_k \eta_k}{\eta_k+\mu_k} R_k(t)u \right) ,$$

where t in $R_k(t)$ is not a dummy variable but the initial point of the time step under consideration. Since $\eta_k/(\eta_k+\mu_k)$ is the

probability that a particular type k target is visible and λ_k the rate of acquisition given continuous visibility, (6.9) is reasonable within the context of the other computations.

Finally, to compute $d_k(u)$ the model employs the equation

$$(6.10)\quad d_k(u) = 1-\left[1 - \frac{r_k}{\mu_k+\rho_k-r_k}\left(e^{-r_k u} - e^{-(\mu_k+\rho_k)u}\right)\right]^{R_k(t)} ,$$

where

$$r_k = \frac{\lambda_k \eta_k}{\eta_k+\mu_k}$$

is interpreted as above and where $\rho_k = \rho_k(B(t))$ is the rate at which *other* shooting weapons are destroying a given target of type k (details of the computation of which may be found in [14]). As previously noted, we believe it would be a reasonable and consistent simplification to take $\rho_k = 0$.

To compute the destruction probability h_j the equation used is

$$(6.11)\qquad h_j = \frac{\alpha_j}{\alpha_j + \mu_j + \rho_j} ,$$

which seems entirely reasonable: once begun, the engagement ends in destruction of the target at rate α_j, in loss of line of sight at rate μ_j and in destruction of the target by another shooting weapon at rate ρ_j.

To complete the calculations necessary to use (6.6), it remains to compute $\overline{G}(x)$ for each $x \in E$. To do this first write

$$\overline{G}(k_j) = E_{k_j}[T] + E_{k_j}[W]$$
$$\overline{G}(\ell_j) = E_{\ell_j}[T] + E_{\ell_j}[W] ,$$

where

T = time spent in acquisition and selection,

W = time spent in firing,

and where $E_x[Z] = E[Z|Y_0 = x]$. From (6.8) it then follows immediately that

$$(6.12)\quad E_{k_j}[T] = E_{\ell_j}[T]$$

$$= t_{j-1}d_j(t_{j-1}) \prod_{r=1}^{j-1} [1-H_r(t_{j-1}-t_r)][1-d_r(t_{r-1})]$$

$$+ (1-d_j(t_{j-1}))$$

$$\times \sum_{r=j-1}^{N-1} \int_{(t_r,t_{r-1}]} x \left(\prod_{\substack{s=0 \\ s\neq j-1}}^{r} \left[1-d_{s+1}(t_s)\right] \times \left[1-H_{s+1}(x-t_s)\right]\right) H_j(dx-t_{j-1}) .$$

Finally,

$$(6.13)\quad E_{k_j}[W] = E_{\ell_j}[W]$$

$$= \frac{1}{\alpha_j + \mu_j + \rho_j} ,$$

which follows from (6.11). The expected length of an engagement is independent of the outcome, as (6.13) states. This completes derivation of the quantities needed to employ the basic attrition equation (6.1). The report does not specify how computational difficulties involving numerical aspects of (6.3) - (6.13) are surmounted in the model.

In Appendix E of [14], several detailed methods for calculation of kill rates α_j corresponding to different firing doctrines are presented, namely--

1) Single-shot with Markov fire adjustment, with hit necessary to destroy target,
2) Single-shot with Markov fire adjustment, with possibility that a missed shot destroys the target by killing the associated personnel,
3) Burst fire with time between bursts but no fire adjustment,
4) Area-lethality mechanism.

The reader is referred to [14] and also to [3] for details of these computations.

We conclude this Section with a description of the equations used to compute attrition resulting from support fire; that is, fire by artillery and mortars or by attack helicopters functioning in a support fire role against artillery batteries, air defense sites, and maneuver units. Factors that determine computed attrition are numbers and types of firing weapons, type of ordnance and delivery technique, target type and activity, environment, type of sensor reporting the target, and accuracy and timeliness of the report. Two qualitatively different ordnances are represented; individually targeted (hit-to-kill) ordnance and area-targeted (area effects) ordnance.

For individually targeted ordnance, the attrition equation is

$$\Delta t_j = [f_j \sum_{\ell} k(j,\ell)p(j,\ell)]M \ , \tag{6.14}$$

where

j = target type,
t_j = number of type j targets vulnerable,
Δt_j = attrition to targets of type j,
ℓ = posture class of targets,
$p(j,\ell)$ = fraction of type j targets that are in posture class ℓ,
$k(j,\ell)$ = probability that one unit of expended ordnance directed at a target of type j in posture class ℓ destroys the target,

f_j = probability that a type j target is chosen to be attacked,

M = number of ordnance units expended.

It appears that a computation of the form (6.14) is performed for each weapon functioning in a support fire role and that effects are then summed. It is generally the case that

$$f_j = \frac{t_j}{\sum_k t_k} ,$$

whereupon (6.14) is of independent engagement initiation form. Applicability of (6.14) to individually targeted ordnance is then possibly inappropriate, especially in terms of underlying assumptions; the reader is referred to [7,11] for further details.

For area-targeted ordnance it is assumed that each target (a maneuver unit, say) is partitioned into one or more equivalent subtargets, no two of which can be damaged by a single attack. Attacks against the entire target are allocated uniformly among the subtargets. To compute, for a given subtarget, the attrition to target elements comprising that subtarget, the VECTOR-2 model uses the equation

$$\Delta t_j = t_j[1 - (1-d_j)^N] , \tag{6.15}$$

where

j = type of target element,

t_j = number of target elements of type j in the subtarget,

Δt_j = attrition to type j target elements in the subtarget,

d_j = fractional damage to one target element of type j in one attack on entire subtarget,

N = number of attacks on subtarget.

As appropriate, the number of attacks, N, is the number of artillery volleys fired at a given subtarget or the number of aircraft (in the CAS mission) attacking the subtarget. To

compute the fractional damage d_j resulting to one target element from one attack, the VECTOR-2 model uses the equation

$$(6.16) \qquad d_j = \sum_{k=1}^{M} (-1)^{k-1} \frac{\dfrac{c_j^2 + s^2}{q^2}}{\dfrac{c_j^2 + s^2}{q^2} + k} \frac{(M)_k}{k!} \left(\frac{c_j^2}{c_j^2 + s^2} \right)^k ,$$

where

M = number of firing "patterns" constituting one attack,

q = circular error probability of center of pattern around subtarget,

s = subtarget size parameter,

c_j = lethality radius of one pattern against target element of type j,

and where $(M)_k = M(M-1) \cdots (M-k+1)$.

Equation (6.16) is based upon target elements being uniformly distributed over the area comprising the subtarget and on normally distributed delivery errors; cf. [14] for details.

For small values of the d_j, equation (6.15) becomes

$$\Delta t_j \sim d_j t_j N ,$$

which is an attrition equation of proportional engagement initiation form, whose applicability to "area-targeted" ordnance is uncertain; cf. [7,11].

The reader should note that only resource groups in the first band of zones across the FEBA are vulnerable to artillery; in particular, usable air bases are not vulnerable to artillery.

7. ATTRITION IN AIR COMBAT

In this Section we describe the methodology employed in VECTOR-2 for computing losses in air-to-air interactions, ground-to-air interactions, and air-to-ground interactions other than those involving maneuver units. Compared to that used for assessing the effects of ground combat interactions, the air combat methodology is both simple and simplistic, possessing neither the overwhelming level of detail nor the degree of sophistication of the former. The air-to-air combat model is structured on the basis of air groups, which are constructed and have missions assigned to them by the tactical decision rules discussed in Section 5. Construction of air groups and assignment of missions seem to be done on the basis of sectors; however, time evolution of the location and status of each group is represented individually. Air-to-air combat involving a flight of attacking aircraft and associated escorts will occur if enemy target acquisition resources acquire the attacking air group, if a flight of interceptors is allocated (as determined by tactical decision rules) to attempt an intercept, and if the attempted intercept is successful. It is not clear how success or failure of an attempted intercept is determined.

Let us now consider an engagement involving attacking aircraft, escorts, and interceptors under the assumption of a successful intercept. Let

A_i = number of attacking aircraft of type i,
E_j = number of escort aircraft of type j,
I_k = number of interceptors of type k,

denote the respective numbers of aircraft involved. Once the engagement is initiated, movement of aircraft involved ceases until engagement effects are calculated, which is a reasonable procedure. An air-to-air engagement consists of one or more long-range phases involving stand-off weapons followed by a short-range duel phase. During each phase targets are selected randomly from those acquired and simultaneous engagement of more than one target is permissible. After each phase of the engagement, either side can disengage (using tactical decision rules). For disengaging penetrators, the mission is considered aborted and aircraft turned back toward their bases, apparently unable to attempt to resume their original mission. If interceptors disengage, penetrators continue toward their targets. Engagements can occur involving outbound as well as inbound attacking aircraft but, unreasonably, appear to be treated identically.

A single form of attrition equation is used to compute aircraft losses to both sides in all phases of air-to-air combat. While this is consistent, it also seems an undesirable and unnecessary oversimplification. In particular, it fails to account for asymmetry of the objectives of interceptors and penetrators, which is represented, for example, in the barrier penetration attrition model described in [1]. According to the VECTOR-2 equation, the number of attacking aircraft of type i destroyed in a given phase of engagement, ΔA_i, is given by

$$\Delta A_i = \frac{A_i}{A + E} \sum_k a(i,k) \min\{A_i, m_k\} I_k , \tag{7.1}$$

where

$a(i,k)$ = "attrition caused by one interceptor of type k that engages only attacking aircraft of type i",

m_k = maximum number of aircraft that an interceptor of type k can engage simultaneously,

and where

$$A = \sum_i A_i$$

and

$$E = \sum_j E_j$$

are the total numbers of attacking aircraft and escorts, respectively. The parameters a(i,k) and (possibly) m_k depend upon the phase of the duel. Equation (7.1) is, for large numbers of targets, of independent engagement initiation form (in the taxonomy of [11]) for if $A_i \geq m_k$ for all k, then (7.1) becomes

$$\Delta A_i = \frac{A_i}{A + E} \sum_k a(i,k) m_k I_k \; ; \tag{7.2}$$

the factor $A_i/(A+E)$ represents a uniform fire allocation, cf. [7]. The a(i,k) are only vaguely defined in [14], in the words quoted following (7.1), which make them seem kill potentials of the usual undefinable and uncomputable sort. Observe also that (7.1) allows multiple engagements only against attacking aircraft of the same type, an unwarranted and restrictive assumption.

Use of an independent engagement initiation attrition equation to represent air-to-air combat involving large numbers of targets seems reasonable even though the fire allocation in (7.1) represents fairly precise coordination and information transfer by the defense. When engagement capabilities of interceptors become unlimited (i.e., $m_k \to \infty$ for all k), then (7.1) becomes an attrition equation with proportional engagement initiation:

$$\Delta A_i = A_i \left(\frac{A_i}{A+E} \right) \sum_k a(i,k) I_k \; , \tag{7.3}$$

which also is plausible. Note that (7.3) obtains for small numbers of attacking aircraft, in which case engagement opportunities become constrained by the numbers of available targets.

In [13] the reviewer derives from explicit physical and probabilistic assumptions an attrition equation to which (7.1), (7.2), and (7.3) can be regarded as linear approximations; the interested reader is referred there for further details. Another derivation appears in [6].

For completeness we observe that losses ΔE_j of escorts of type j are given by

$$\Delta E_j = \frac{E_j}{A + E} \sum_k a'(j,k) \min\{E_j, m_k\} I_k \ , \tag{7.4}$$

where a'(j,k) is the attrition caused by one interceptor of type k that exclusively engages escorts of type j. Similarly, losses ΔI_k of interceptors of type k are given by

$$\Delta I_k = \frac{I_k}{I} \left[\sum_i \tilde{a}(k,i) \min\{I_k, \tilde{m}_i\} A_i + \sum_j \hat{a}(k,j) \min\{I_k, \hat{m}_j\} E_j \right] , \tag{7.5}$$

where I is the total number of interceptors, the $\tilde{a}(k,i)$ and $\hat{a}(k,j)$ are attrition potentials, and the $\tilde{m}_i$ and $\hat{m}_j$ are maximum numbers of simultaneous engagements.

While (7.1), (7.4), and (7.5) are not individually unreasonable, together they have several deficiences. First and possibly most important is that they ignore differing objectives of attacking aircraft, escorts, and penetrators, unless one represents them by the attrition potentials. Second, consideration of time sequencing of events seems to be ignored; in particular it seems that interceptors would often encounter escorts before attackers, so that only interceptors surviving interactions with escorts should be allowed to engage attacking aircraft. This coarse treatment of air-to-air combat is a disappointment.

Next we consider losses to ground weapon systems of aircraft on overflight (i.e., aircraft whose targets are not immediately defended by the ground weapon systems). This portion of the VECTOR-2 model is even more simplistic than the representation of air-to-air combat, but still seems reasonable: losses on overflight probably do constitute a nonnegligible but not significant component of aircraft attrition.

Aircraft on overflight are fired upon by air defense weapons provided that--

1) the aircraft be acquired by the air defense weapons;
2) the air defense weapons not be engaging targets of higher priority (aircraft attacking either the weapons themselves or a target they defend);
3) the aircraft not be in an intercept corridor (in which interceptor aircraft are also present);
4) ordnance be available.

Inputs to the attrition calculation are firing rates for different types of air defense weapons; other quantities appearing below are internally computed.

Below, the indices "g" and "h" denote groups of attacking aircraft, while "j" and "k" denote types of attacking aircraft. Let

$A(g,j)$ = number of type j aircraft in group g,

D_i = number of type i air defense sites to which aircraft are vulnerable,

$\beta(j,i)$ = *rate* at which one type i air defense site destroys aircraft of type j, given continuous engagement,

Δt = time interval under consideration (Level 1).

The $A(g,j)$, D_i, and the time interval Δt are calculated within the model, the first two from explicit positional data. Losses of attacking aircraft of type j in group g, $\Delta A(g,j)$, are then given by

$$\Delta A(g,j) = \left[\sum_i \beta(j,i)e(g,j,i)D_i\right]\Delta t \ , \tag{7.6}$$

where the e(g,j,i) are allocation factors computed according to the equation

$$(7.7) \quad e(g,j,i) = \frac{p(g,j,i)A(g,j)}{\sum_{h,k} p(h,k,i)A(h,k)} \left[1- \prod_{h,k} (1-p(h,k,i))^{A(h,k)}\right] .$$

In Equation (7.7), p(g,j,i) is the proportion of time an aircraft of type j in group g is in range of and acquired by a (representative) air defense side of type i; it is computed within the model from data concerning aircraft flight paths and "locations" of air defense sites. Observe that with i fixed

$$\sum_{g,j} e(g,j,i) = \left[1- \prod_{h,k} (1-p(h,k,i))^{A(h,k)}\right]$$

so that (7.6) and (7.7) account for weapons with no targets in range and acquired. Implicit in (7.7) is the uniform fire allocation represented by the first factor on the right-hand side. This assumption is not unreasonable; the further complications involved in more detailed schemes seem unnecessary.

Finally we discuss aircraft-related combat effects in the target area; specifically, since air-to-ground losses are treated in Section 6 on attrition in ground combat, we described computation of losses of attacking aircraft inflicted by air defense sites in the immediate vicinity of the target. The target may be an air defense site. The following steps constitute this interaction.

1) An aircraft group arriving in the vicinity of its preassigned target must reacquire the target in order to attack it. If reacquisition is unsuccessful, an attempt may be made to acquire a specified secondary target (which seemingly must be nearby). If no target is acquired, the attack is aborted.
2) If a target is acquired, all aircraft (except escorts) in the group attack it. Escorts are not vulnerable to target area defenses.

3) Attacking aircraft make one or more passes over a target. At the start of each pass, air defense weapons fire upon the aircraft, causing attrition that is calculated in equation (7.8) below. The attacking side may abort the attack at the end of any pass if losses are excessive. Each successful pass by one aircraft results in delivery of a specified type and amount of ordnance, the effects of which are described in Section 6.

The following equation is used in VECTOR-2 to compute losses of aircraft to target area defenses on *one pass* over the target. Let

$\tilde{A}_j$ = number of aircraft of type j that survive to make the pass over the target,

D_i = number of type i air defense sites defending the target,

$t(j,i)$ = length of time in one pass that an aircraft of type j is acquired by an air defense site of type i,

$\gamma(j,i)$ = *rate* of kill given continuous acquisition and engagement of type j aircraft by type i air defense site.

The $\gamma(j,i)$ are inputs to the model; the remaining quantities above are calculated and dynamically updated within the VECTOR-2 model. Losses of type j aircraft on one pass, $\Delta\tilde{A}_j$, are then given by the equation

$$\Delta\tilde{A}_j = \frac{\tilde{A}_j}{\sum_k \tilde{A}_k} \sum_i \gamma(j,i)t(j,i)D_i \ . \tag{7.8}$$

As noted earlier in this Section and in [11], this attrition equation is of independent engagement initiation form and, consequently, seems reasonable in this context.

8. MOVEMENT OF GROUND UNITS AND AIRCRAFT

In terms of representation of FEBA location and movement, VECTOR-2 is clearly superior to the other models. Unlike the IDAGAM I, Lulejian-I, and CONAF Evaluation models, the VECTOR-2 model does not contain a FEBA movement computation *per se*; in particular, FEBA movement is *not* computed using functions based on historical data whose argument is a force ratio. Rather, movement of ground force maneuver units is represented explicitly and computed positions are updated each Level 1 time period. FEBA position is simply the line of farthest current advance of front line maneuver units and changes over time as maneuver unit locations vary. Therefore, FEBA position is determined internally from explicit geographical locations of resources, rather than--as in the other models--by an externally and artificially imposed function of force ratios.

To avoid ambiguities in possible definition of the FEBA and because of an inherent difficulty in representing breakthroughs and encirclements (which are physically identical but interchange the roles of attacker and defender), the latter are not allowed. Even with this restriction, VECTOR-2 remains much more realistic and detailed than the three other models.

To obtain the current location of the FEBA (for each side--the sides are generally separated by a positive distance as described in Section 3), it suffices to observe the positions of front line maneuver units. Evolution of these positions, in turn, occurs subject to the following assumptions:

1) Reorganizational movement occurs instantaneously at the start of each Level 6 time period if the distances

involved are sufficiently short; for more extended movements a period of unavailability is represented.

2) Tactical movement of maneuver units is computed in terms of the coordinate geography and occurs in straight lines except when (in movement of a reserve unit) it would be necessary to cross the FEBA.

3) Movement occurs at input-prescribed rates that are functions of weather, terrain, and possible presence of a minefield.

4) During close combat engagements, effects of range and movement are represented by variations in parameters defining different line of sight processes. This dependence is not described explicitly in [14] but some aspects of it are treated in [3].

FEBA movement is computed and available as an output for each front line combat arena. For each side, FEBA movement is by definition the physical movement of the front line force in that arena.

Movement of ground resource groups other than maneuver units is not included in the model; such resource groups are assumed to possess sufficient capability to maintain their prescribed distances from the FEBA (cf. Section 3).

Location and movement of air groups, namely attack and escort groups on the attacking side and interceptor groups on the defending side, is--like maneuver unit location and movement--represented explicitly in terms of coordinate battlefield geography and updated each Level 1 time period. The following assumptions are in force during these calculations:

1) Each air group is represented by a single coordinate position.

2) Flight paths consist of straight line segments transversed at input-specified speeds. Flight paths are constructed and selected by tactical decision rules that assign aircraft missions.

3) An air-to-air or air-to-ground engagement stops progress along a flight path for the duration of the engagement.

Flights of attack helicopters functioning in the support fire role (rather than as maneuver unit weapons) are treated in the same way.

A significant problem in theater level combat modeling is determination of *engagement eligibility*: what targets are vulnerable to what shooting weapons? Some models, because of imprecise rules for this determination, seem to calculate unreasonably high losses. VECTOR-2, on the other hand, can deal easily and realistically for the most part with engagement eligibility by calculating it from explicit positional data. But when some relevant locations, such as those of air defense sites, are notional, whereas other locations, such as those of overflying aircraft, are explicit, the methodology encounters the same problems that plague other models.

9. INTELLIGENCE AND TARGET ACQUISITION

The important physical phenomena of intelligence and target acquisition are represented in more detail and more realistically in VECTOR-2 than in the CONAF Evaluation, IDAGAM I, and Lulejian-I models. In the latter models, target acquisition is generally represented, if at all, only in terms of ill-defined probabilities of detection. Target intelligence is usually nonexistent and weather intelligence is not present since weather itself is not represented. Target intelligence and acquisition for weapons in maneuver units have been treated in detail in Section 6 and will not be discussed further here. After some preliminary remarks, however, we will discuss target acquisition for firing weapons in support fire roles.

Intelligence represented in VECTOR-2 is of four kinds:

1) Weather intelligence as user-input five day weather forecasts for each side;
2) Target intelligence for maneuver units, air defense weapons, and aircraft, as previously described in Sections 6 and 7;
3) Target intelligence for allocation of support fire, as expected numbers of acquisitions by observation resources, expected location errors, and expected times since acquisition;
4) Enemy order-of-battle intelligence, as Bayesian estimates of numbers of ground targets by type and zone (cf. Section 3) and numbers of aircraft by sector.

The third and fourth categories will be discussed in more detail below.

Intelligence concerning terrain and enemy posture are not represented in the model; both are assumed to be perfect. Alternative assumptions may be possible through particular sets of tactical decision rules.

Concerning target acquisition for support fire (by artillery, helicopters or aircraft), the fourteen types of observation resources are partitioned into two classes:

1) Observation resources that do not adjust fire but simply report, subject to error, the location of an acquired target and then resume search efforts;
2) Observation resources that adjust fire and attempt to maintain acquisition, with negligible error, until the fire is delivered.

The following enemy resource groups are vulnerable to acquisition:

1) Maneuver units
2) Artillery
3) Air defense sites
4) Command posts
5) Logistic targets
6) Air bases
7) Enemy target acquisition resources
8) Aircraft in flight
9) Two user-specified classes of resource groups.

Acquisition of a target requires coverage of the target (it must be in range and in the field of view of the observation resource) and existence of a line of sight. Acquisition information is retained in the model until support fire is allocated to the target or the target becomes invisible, or until the information becomes too old to be reliable (the target can move). It is further assumed that there is neither acquisition of false targets, nor misidentification of targets, nor cueing of one observation resource by another.

Subject to these assumptions, the probability that a particular observation resource acquires a particular target in t time units or less is given by

$$p(t) = 1 - \left[1 - p_C p_L (1-e^{-\lambda t})\right]^S , \tag{9.1}$$

where

$p(t)$ = probability of acquisition in $[0,t]$,

S = number of sensors comprising observation resource,

p_C = probability of coverage for each sensor,

p_L = probability of existence of line of sight for each sensor,

λ = acquisition rate given coverage and existence of line of sight.

The latter three quantities are computed within the model from data concerning relevant positions, terrain, and weather; details are not given in [14].

Searches are conducted by different sensors in a user-prescribed order; previously acquired targets are not reacquired. Hence each sensor decreases the set of unacquired targets by acquiring targets undetected by prior searches; in this way [14, p. D-2] acquisition data are "reported...by the most preferred sensor system able to achieve detection."

Moreover, target acquisition entails processing time and possible communication delays, so that it is possible that a sensor system may become saturated and unable to make further detections or reports.

Outputs of this process, namely expected numbers of acquisitions of targets against which support fire has not previously been allocated, together with location errors and times since acquisition, become inputs to the equations that calculate attrition effects of support fire.

The Bayesian scheme for updating enemy order-of-battle intelligence is, for a single type of target, of the form

$$\hat{n}_c = p(t)n_c + (1-\hat{p}(t))\hat{n}_p \ , \tag{9.2}$$

where t is the length of the time interval under consideration, $p(t)$ is given by (9.1), and

$\hat{n}_c$ = estimated current number of targets,

n_c = actual current number of targets,

$\hat{p}(t)$ = estimated acquisition probability,

$\hat{n}_p$ = previous estimate of number of targets.

This equation is reasonably realistic and adequately simple.

10. COMPUTER-RELATED ASPECTS

In this Section we briefly describe some computer and data requirements and features of the VECTOR-2 model. Our treatment is neither complete nor intense; the interested reader is referred to [15] for details.

Requirements of VECTOR-2 for input data, computer core, and computer running time are large, often exceeding significantly (by a factor of 2 or more) analogous requirements of the CONAF Evaluation, IDAGAM I, and Lulejian-I models. To some extent severity of these requirements is mitigated by ancillary computer programs such as the data preprocessor described below; however, even if all the reductions discussed below were effected, we believe that VECTOR-2 would continue to require more inputs, more core, and more running time than the other three models.

Since it is essentially an immutable characteristic of the model, the core requirement of

1) 150,000 32 bit words for computations involving each sector, and
2) 50,000 32 bit words for theater-level calculations

is difficult to change. The computer program is structured, however, so that sectors are computationally dependent only through theater-level calculations, which makes simultaneous consideration of 2 or more sectors never necessary. Some reduction in core requirements might be achieved by decreasing the numbers of weapon types or combat arenas, but is not likely to be significant.

Input data requirements are approximately:

1) Data concerning theater-level performance, tactics, and terrain variables (about 9500 values);
2) Data concerning sector-level performance, tactics, and terrain variables (about 21500 values);
3) Initial data, force arrivals, and weather information arrivals (about 1500 values);
4) Run control parameters (about 50 values).

One also must regard the 34 tactical decision rules as required inputs; the user wishing to derive these latter inputs faces a substantial burden in terms of analysis and programming (the tactical decision rules must be FORTRAN subroutines compatible with the main program). However, VECTOR-2 is based on physically measurable performance data; it does not entail the same amount of data preparation (as opposed to number of data values) as do the other models; for the latter, inputs such as "kill potentials" or "kill probabilities" must be calculated (basically by hand) from more primitive, often unspecified, data. Often, there is no suggested, let alone justified, method of making calculations.

Furthermore, the physically measurable inputs to VECTOR-2 can be prepared automatically by use of a *Data Preprocessor*, which extracts these inputs from a data base maintained by OSD. The Data Preprocessor is able to prepare inputs representing weapons system performance, order of battle, logistics, and communications. The user still must manually prepare data related to environment and scenario and to force numbers and arrivals, and also make any changes needed in automatically prepared data.

We emphasize that the greater input data requirement of VECTOR-2 is in itself neither an advantage nor a disadvantage of the model; it is a *difference* between VECTOR-2 and the other models. Some of the comments above depict it as a disadvantage; However, it is an advantage to the extent that more--and

physically definable--phenomena affect the results of the model. Certainly the input requirement is consistent with the level of detail of the model and perhaps is best judged (if judgment is the objective) in that larger context: if the level of detail is deemed appropriate, the input requirement must be accepted; if the level of detail is unnecessarily high, so is the input requirement.

The computer program for the VECTOR-2 model is very long: 12,000 FORTRAN statements comprising 20,000 lines of code. In order to be more efficient, the main program, including the tactical decision rule subroutines, accepts binary inputs and produces (for every run) a full set of binary outputs. The main program is complemented by a binary data formatter that produces necessary binary inputs and a *Postprocessor* that prepares user-selected output tables from the full set of binary outputs. An advantage of this structure is that, after examination of selected outputs, further outputs of interest can be obtained with relatively little effort; other models might have to be run again.

The available outputs are standard, as the following list demonstrates:

1) Summary outputs provided once per day: resource inventories at FEBA, theater losses, and FEBA position.
2) Theater-level outputs provided once per day: resource strengths by sector, resource losses by type and sector, loss attributions, FEBA positions and movement, intelligence, resource arrivals and allocations to sectors, aircraft and shelter inventories by sector, aircraft allocations to missions.
3) Sector-level outputs provided once per day: weather, map of zones, resource groups by zone and composition, maneuver unit hierarchy, supply distribution and consumption, front line combat effects.
4) Sector-level outputs provided several times per day: resource losses and loss attributions, maneuver unit

losses, target acquisition and support fire allocation, air groups flown, suppression and damage of air defense weapons.

5) Combat arena level outputs provided several times per day: status, inventories, and losses of front line units, attribution of maneuver unit losses, log of events.

A possibly significant property (as yet not fully documented) of the VECTOR-2 model is its running time requirement. The level of detail and shortness of suggested Level 1 time periods (see page 7) imply running times orders of magnitude greater than those of the comparable models. Such running times are not, *a priori*, a deficiency; they should be viewed as part of the price the user must pay for the model's detail in both physical and temporal senses. The report [14] includes no data concerning running times for campaigns on the order of 90 days; it also states that (at the time [14] was prepared) no such runs had been accomplished. If, for a campaign of prescribed duration, running time based on the level sizes suggested in Section 2 is unacceptably high, time steps could be increased. For example, a reduction of 90 percent in running time has been reported to result from the following changes:

CHANGE	FROM	TO
Level 1	30 seconds	15 minutes
Level 2	3 minutes	1 hour
Level 3	15 minutes	2 hours
Level 4	1 hour	2 hours

These changes reduce the daily number of Level 1 evaluations per combat arena from 2880 to 96. It would be worthwhile to perform a careful analysis of the resultant changes in output values.

An extremely useful computer tool is the *Program Change Monitor* described in detail in [15]. This program is designed to aid the user in the generally difficult task of changing the program (e.g., deleting certain variables) and seems to be

an excellent idea. For changing dimensions of arrays, however, alternative methods are available; for example, in IDAGAM I some dimensions (e.g., numbers of weapon types) are themselves inputs to the model (subject to upper bounds of course).

A final, worthwhile feature of the model is its "restart" capability. At the end of each run, final values of evolving state variables are written onto a magnetic tape. Should it then be necessary or desirable to continue that run this tape may be used as input and the previous run is resumed where it left off. There is no need to begin anew.

The reviewer has not analyzed the VECTOR-2 computer program and cannot vouch for consistency with published documentation or numerical aspects.

11. SUMMARY

Because of its level of detail, explicit coordinate geography, and extremely fine time divisions, the VECTOR-2 model is *potentially* a qualitative as well as quantitative advance over other existing models (CEM, IDAGAM I, Lulejian-I, and VECTOR-1). The increased fidelity of representation of space, time, attrition, and movement is impressive, but the word "potentially" is not to be ignored. In particular, the (undoubtedly significant) potential of the model may not be realized for any of the following reasons:

1) The model may not run in an acceptably short length of time when used at the full extent of its capabilities to represent geography and--more importantly--time. With time steps of the same order as those of comparable models, VECTOR-2 becomes less distinguishable from them. To our knowledge it is not yet known whether VECTOR-2 requires impractically large running times.

2) Even if its geographical and temporal representations can be fully exploited, VECTOR-2 may not produce discriminations that will be believed by model users but that could not also be obtained with less complicated, faster, and cheaper models. The prevalent reaction to theater-level models is extreme skepticism; the models are regarded as capable (at best) of making gross qualitative distinctions among force structures, weapon characteristics, and tactics. It seems agreed that no credence should be accorded to actual values of outputs or pairwise comparisons based on nearly identical outputs. Given this (reasonable) attitude of distrust, it

it may be that the believable and usable results obtained using VECTOR-2 might more cheaply, easily, and quickly be obtained using another model.

On the other hand, the VECTOR-2 model may be judged on essentially an intellectual basis as sufficiently more detailed and realistic than the comparable models. As such, (finer) distinctions obtained using it will be believed relative to those of the comparable models. We feel that quantitative results of theater-level models will never be credible in an absolute sense, but VECTOR-2 eventually may be viewed as producing more believable and finer qualitative distinctions. Should this happen, VECTOR-2 will be an order-of-magnitude improvement over the other models in practice as well as potential.

In terms of more specific conclusions, we list below the distinguishing and significant qualities of the VECTOR-2 model. As is often the case with a model, the same qualities are at once its greatest strengths and greatest liabilities. Flexibility, detail and realism, complexity, mathematical sophistication, and so on are strengths only if they are understood, respected, and not abused. Unfortunately, the greater the potential strength, the greater the opportunity for misunderstanding or misapplication, the latter being more user faults than model faults.

The distinguishing and significant qualities of the VECTOR-2 model are as follows:

1) The detailed representation of space, time, movement, and attrition constitutes the greatest single strength of the VECTOR-2 model. There is greater fidelity to the physical reality of combat, and also fewer arbitrary aggregations, fewer arbitrarily imposed sequences of events, and more reliance on definable and measurable inputs. In particular, computation of engagement eligibility and--especially--FEBA movement from

internal and explicit geographical data, represent significant advances over the methods used in the comparable models. Moreover, the VECTOR-2 structure allows for actual rather than artificial attribution of losses, permits dynamic representation of the effects of support fire, and eliminates the artificial front-to-flank ratios present in the other models.

2) The VECTOR-2 model represents (sometimes rather simplistically) phenomena such as target acquisition, communication delays, and imperfect knowledge of opposition forces and intentions that are treated either not at all or very vaguely in other models. While the treatment may not be the best, at least the user is made more aware of limitations of the model and of the arbitrary but necessary assumptions involved.

3) The tactical decision rules in VECTOR-2 permit the user essentially unlimited flexibility and freedom in representing decision making processes relevant to the combat, provided that he or she has the resources to exploit the tactical decision rule structure. Adaptive resource allocation schemes that are impossible in some of the other models are quite feasible with VECTOR-2. Furthermore, even those models (notably CEM) with adaptive decision making structures do not allow the structure itself to be changed from run to run; such changes are possible using VECTOR-2 if the user can produce alternative tactical decision rules. Finally, even if only the demonstration set of tactical decision is used, the structure as a whole serves to isolate and correlate decision making processes in the model; as a consequence, these processes become easier to understand both individually and collectively than in other models.

4) The VECTOR-2 level of detail, in addition to being significantly greater in many respects than those of other models, also generally is consistent internally. Some processes, such as target acquisition and communication delays, are given justifiably simplistic treatments. The exception

(as also in VECTOR-1) is the air combat portion of the model. Although movement is represented in extreme detail, engagements among aircraft and fire of air defense weapons at aircraft are not.

5) The extreme detail and complexity of the model are not without drawbacks in addition to that of running time. There is the difficulty of understanding the model sufficiently well to use it intelligently. Also, while the detail of the model means that none of the arbitrary choices necessary to construct the model has a significant effect in itself, there are many more choices to be made. While the net effect is not certain, it seems that the choices will tend to balance out so that the overall effect of arbitrary choices will be less than in the comparable models.

6) In order to better understand the VECTOR-2 model in both absolute and relative (to other models) terms, it may at some time be useful to consider the following questions.

a) Does a simplified attrition structure materially alter the outputs of the model?

b) What is the effect of increases in the sizes of lower level time steps?

c) Does a more complicated air model that is more consistent with the ground portion of the model (and includes, for example, a more realistic variety of aircraft missions) change the behavior of the model as a whole?

d) Can a simplified, fast-running version of the model, which ignores, for example, weather or communication delays and imperfect target acquisition, or combines acquisition and line of sight processes, be devised? How and to what extent does the simplified version differ from the fully detailed version of the model and from CEM, IDAGAM I, and Lulejian-I?

7) Except that it does not describe the demonstration package of tactical decision rules, the model documentation [14,15, 16] is excellent in terms of completeness and clarity. It has made this review feasible.

If there is a negative conclusion to be drawn, it is that VECTOR-2 is not really a theater-level model, although it may be the best corps-level model. In support of this, one may cite the oversimplified air model and the data and running time requirements entailed by use of VECTOR-2 in theater-level analyses analyses. As a purely mathematical structure, the VECTOR-2 model is without question more flexible, general, and realistic than the CONAF Evaluation, IDAGAM I, and Lulejian-I models. Whether practical use can be made of these advantages, though, is uncertain.

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